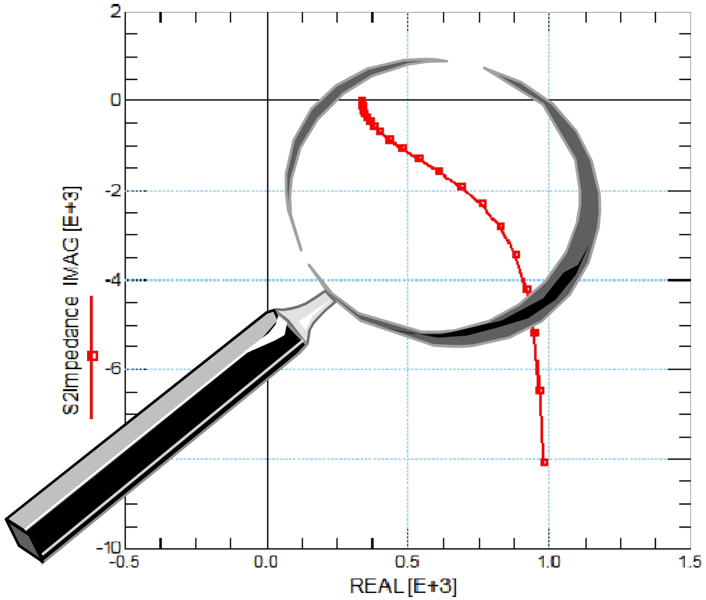
Successful Device Modeling from Impedance Plots

- A Practical Lab Note Book -



Franz Sischka, Nov. 2018

www.SisConsult.de

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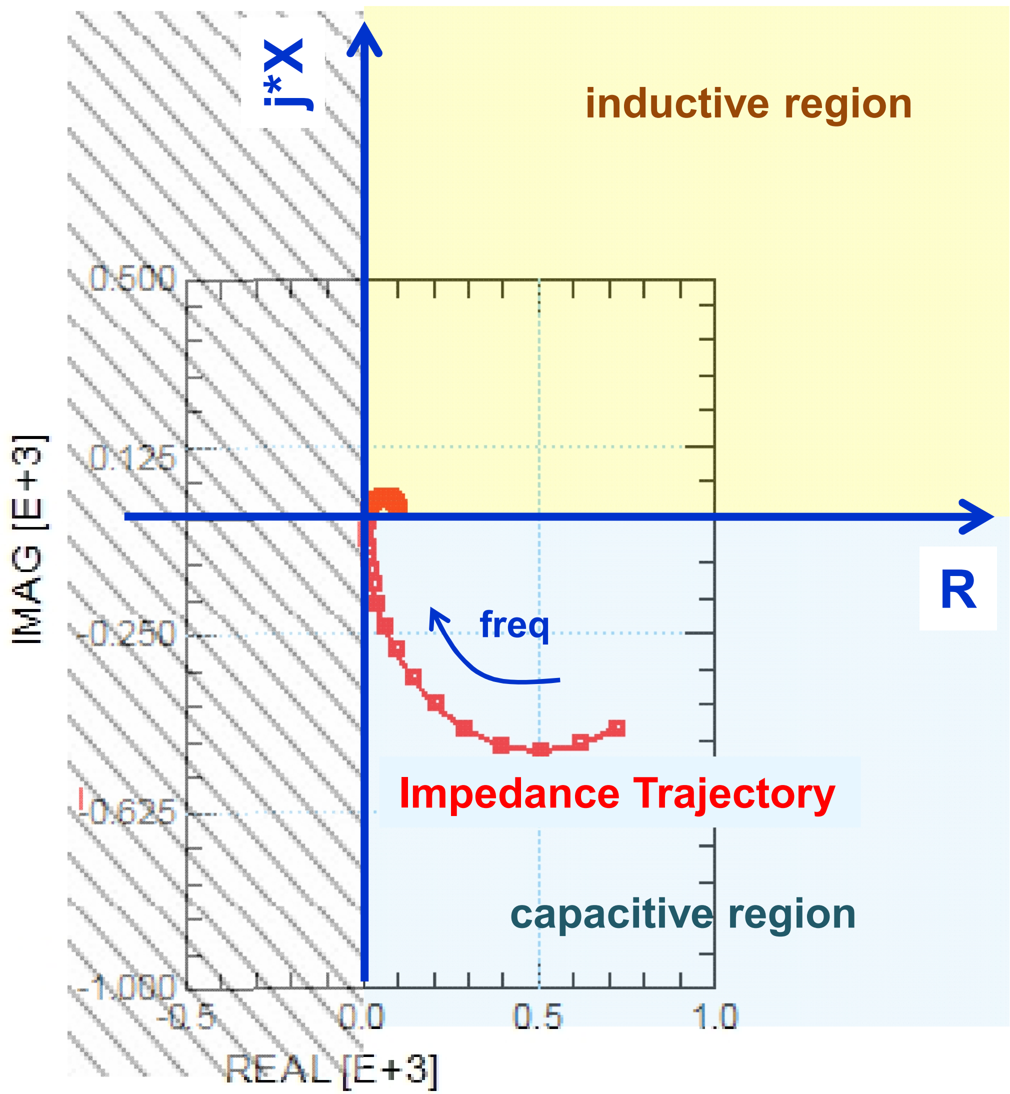
Outline

- **The Impedance Plane Z = R + j*X and Typical Impedance Traces**
- **Impedance Plots from LCRZ Meters**
- **Impedance Plots from S-Parameters**

The Impedance Plane $\underline{Z} = R + j_* X$



The frequency-dependent impedance locus curves (trajectories) can be measured by LCRZ meters, or obtained from S-Parameters

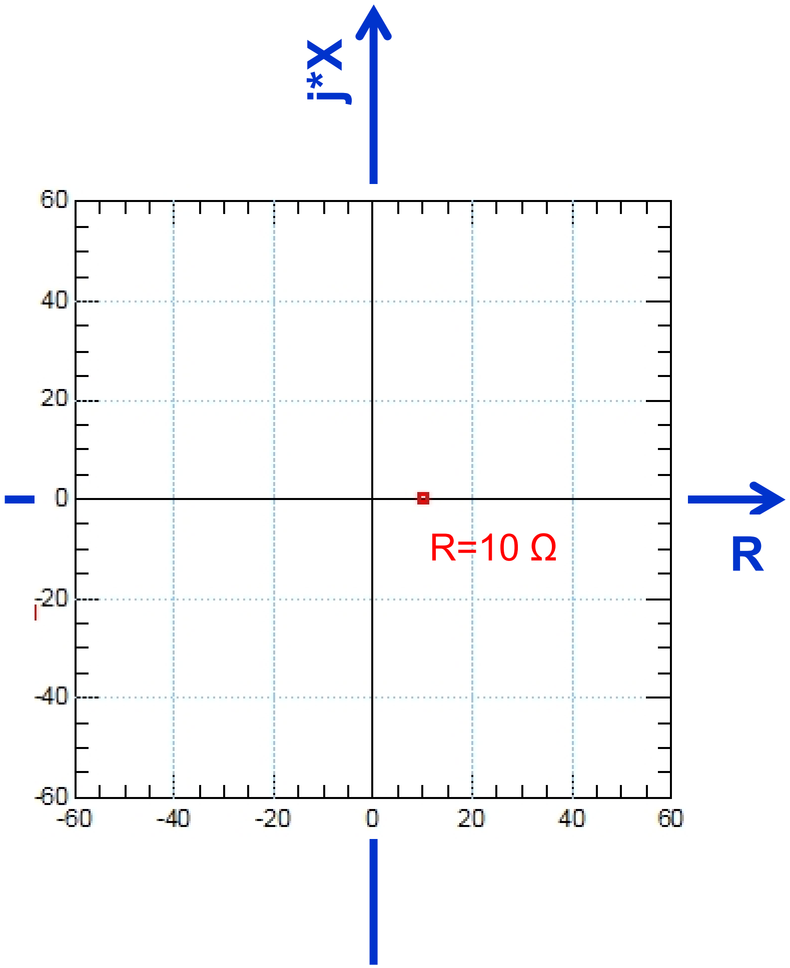
For 2-Port Impedances,

with increasing frequency,
all impedance trajectories turn clock-wise

only the right half-plane is used
(otherwise: $R < 0\ \Omega$!!)

Impedance Examples

Ideal Resistor

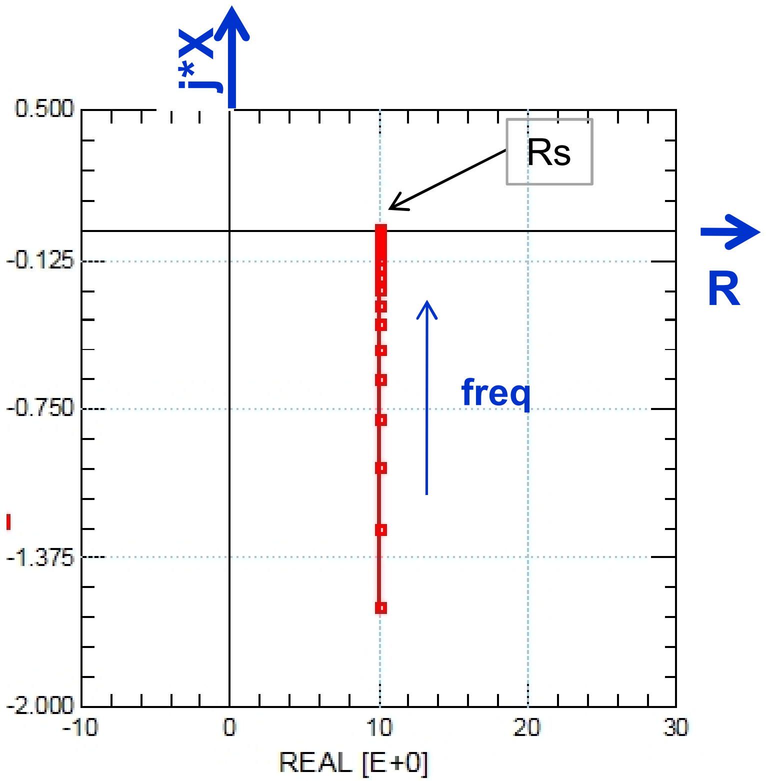



```
Schematic:
*==============================
*
*   1 o---|‾‾|---o 2
*          R
*==============================
```

The impedance is represented by a single point, for all frequencies, on the x-axis of the impedance plot

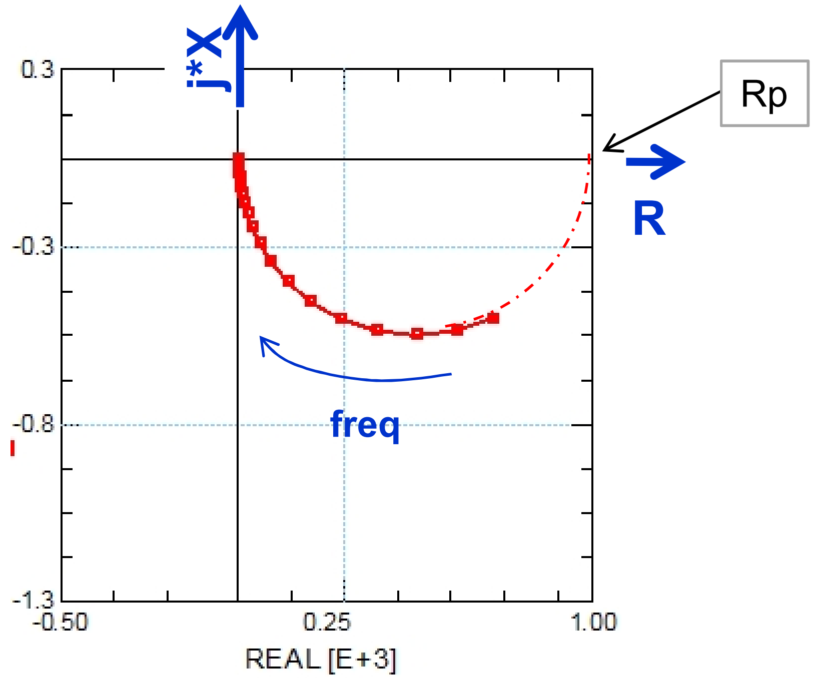
Resistor and Capacitor

```
Schematic:
*===================================
*                   __
*   Port1 o---||---|__|---o Port2
*             Cs     Rs
*===================================
```



Impedance Plot Explanations:
freq -> 0: it's an OPEN: Z=-j*infinite
freq -> infinite: Cs is a SHORT, Z=Rs
in between: with increasing freq, a straight line bottom -> up to Z=Rs

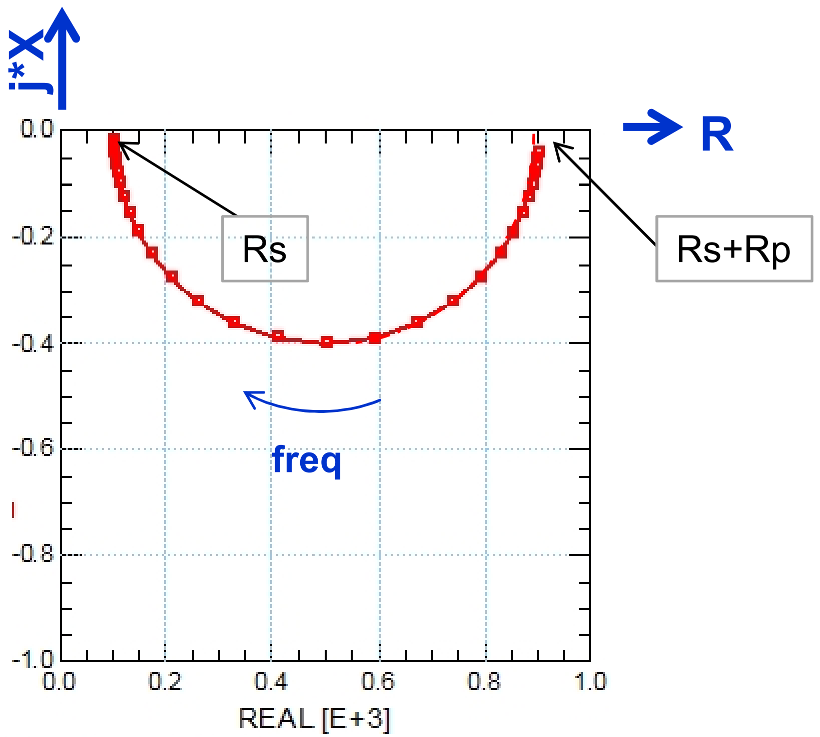
```
Schematic:
*===================================
*              ---||---
*             |   Cp   |
*   Port1 o---o        o---o Port2
*             |   __Rp |
*              --|__|--
*===================================
```



Impedance Plot Explanations:
freq -> 0: Cp is an OPEN, Z=Rp
freq -> infinite: Cp shorts Rp (Z=0)
in between: a half-circle turning from Rp clock-wise (with increasing freq) to Z=0

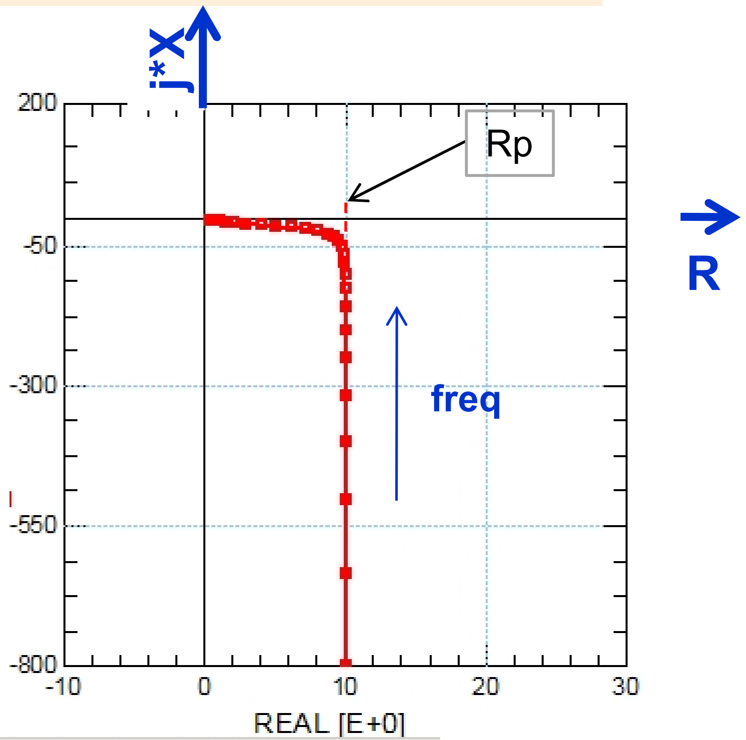
Resistors and Capacitors

```
Schematic:
*==========================================
*                          __Rp
*                       --|__|--
*                      |        |
*               __
*  Port1 o---|__|---o---||---o---o Port2
*             Rs          Cp
*==========================================
```



Impedance Plot Explanations:
freq -> 0: Cp is an OPEN: Z=Rp+Rs is at the right x-axis
freq -> infinite: Cp shorts Rp: Z=Rs
in between: a half-circle turning from Rs+Rp clock-wise (with increasing freq) to Z=Rs

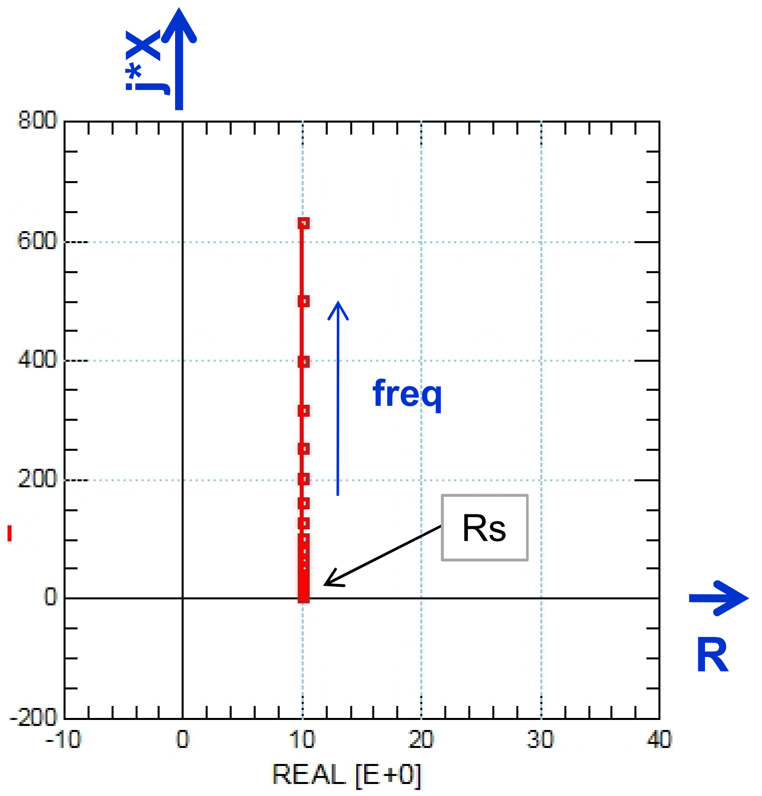
```
Schematic:
*=====================================
*                  ----||----
*                 |     __Cp  |
*  Port1 o---||---o---|__|---o Port2
*            Cs        Rp
*=====================================
```



Impedance Plot Explanations:
freq -> 0: it's an OPEN: Z=-j*infinite
freq -> infinite: Cs and Cp are SHORTs, Z=0
in between:
- if Cs>>Cp: with increasing freq,
  a straight line bottom -> up to Z=Rp,
  then turing to Z=0 by a half-circle
- if Cs<<Cp: with increasing freq,
  a straight line bottom -> up to Z=Rp,
  then turning straight to Z=0

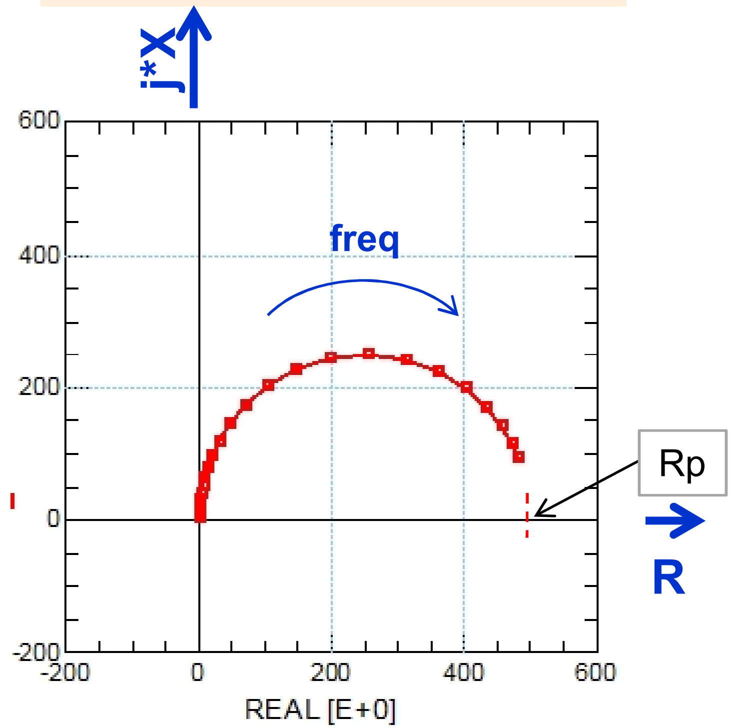
Resistor and Inductor

```
Schematic:
*====================================
*
*   Port1 o---\/\/---|‾‾|---o Port2
*             Ls       Rs
*====================================
```



Impedance Plot Explanations:
freq -> 0: Ls is a SHORT: Z=Rs
freq -> infinite: Ls is an OPEN, Z=j*infinite
in between: with increasing freq, a straight line bottom -> up from Z=Rs to j*infinite

```
Schematic:
*====================================
*              --\/\/--
*             |   Lp   |
*   Port1 o---o        o---o Port2
*             |   __Rp |
*              --|__|--
*====================================
```



Impedance Plot Explanations:
freq -> 0: Lp is a SHORT: Z=0
freq -> infinite: Lp is an OPEN, Z=Rp
in between: a half-circle turning from Z=0 clock-wise (with increasing freq) to Z=Rp

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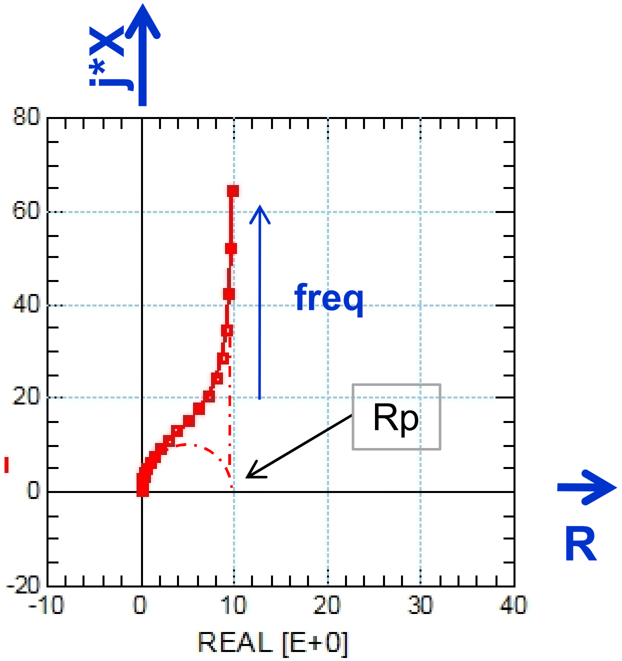
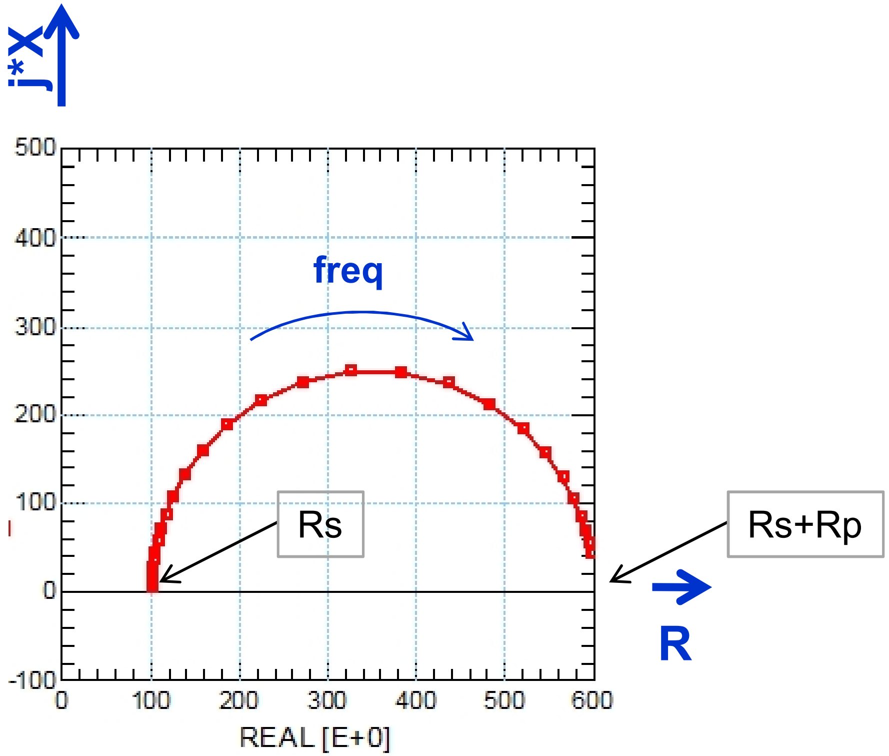
Resistors and Inductors

```
Schematic:
*=====================================================
*                    --\/\/--
*                   |    Lp    |
*  Port1 o---\/\/---o          o---o Port2
*            Ls     |    __Rp  |
*                    --|__|--
* Note: this schematic is typical for spiral inductors
*  incl. skin effect
*=====================================================
```



Impedance Plot Explanations:
freq -> 0: Lp and Ls are both SHORTs: Z=0
freq -> infinite: LS is an OPEN, and Lp too: Z=j*infinite
in between: for typically Lp > Ls: first Lp becomes an OPEN,
while Ls is still a SHORT: half-circle from Z=0 to Z=Rp.
With higher freq, Ls becomes an OPEN too: new end point: Z=Rp+j*infinite

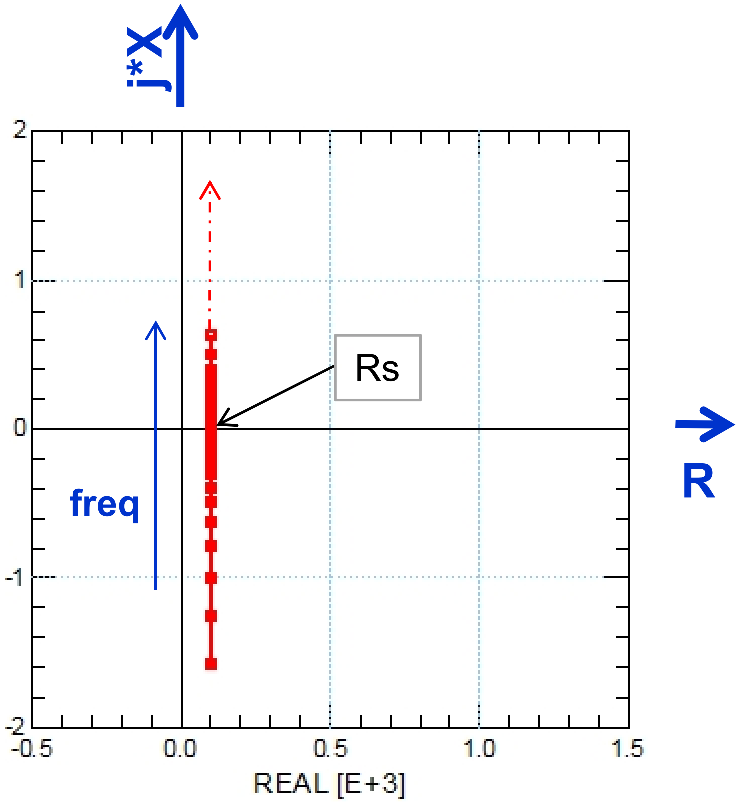
```
Schematic:
*=====================================================
*                     --\/\/--
*            __      |   Lp    |
*  Port1 o---|__|---o          o---o Port2
*            Rs      |   __Rp  |
*                     --|__|--
*=====================================================
```



Impedance Plot Explanations:
freq -> 0: Lp shorts Rp: Z=Rs
freq -> infinite: Lp is an OPEN: Z=Rs+Rp
in between: a half-circle turning from Rs clock-wise (with increasing freq) to Z=Rs+Rp

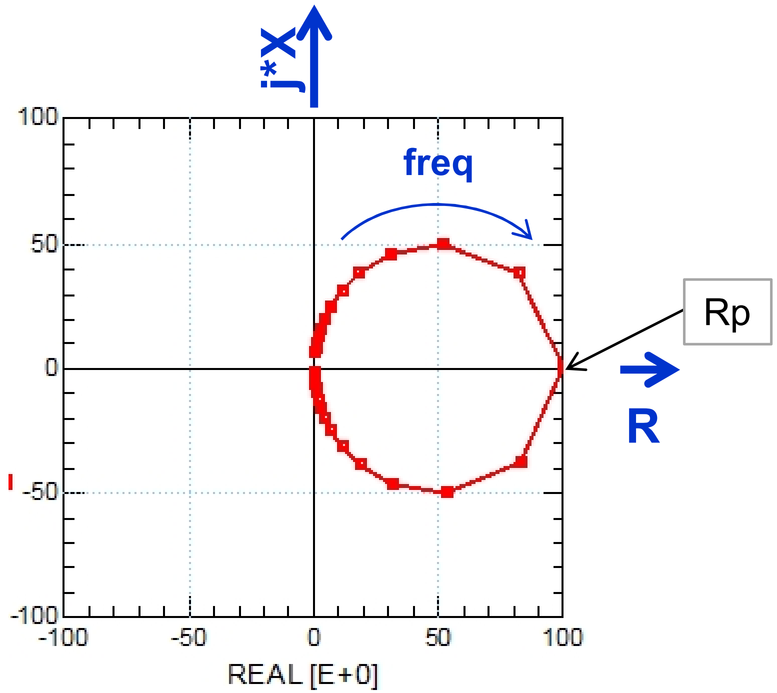
Resonance Circuits

```
Schematic:
*============================================
*
*   Port1 o---|__|---o---||---o---/\/\---o Port2
*             Rs         Cs        Ls
*============================================
```



Impedance Plot Explanations:
freq -> 0: Cs is an OPEN: Z=-j*infinite
freq -> infinite: Ls is an OPEN: Z=j*infinite
in between: resonance: Z=Rs
with increasing freq, a straight line bottom -> up from Z=-j*infinite, towards Z=Rs (resonance), and the further up to Z=j*infinite

```
Schematic:
*=====================================
*                  __Rp
*             ---|__|---
*            |    Lp    |
*  Port1 o---o---/\/\---o---o Port2
*            |    Cp    |
*             ----||----
*=====================================
```



Impedance Plot Explanations:
freq -> 0: Lp is a SHORT, Z=0
resonance: the x-axis is crossed at Rp
freq -> infinite: Cp is a SHORT, Z=Rp

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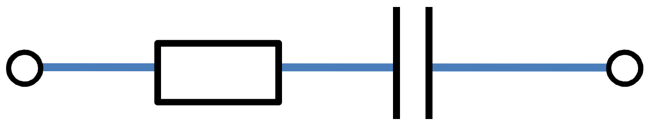
Outline

- The Impedance Plane Z = R + j*X
  and Typical Impedance Traces
- **Impedance Plots from LCRZ Meters**
- Impedance Plots from S-Parameters

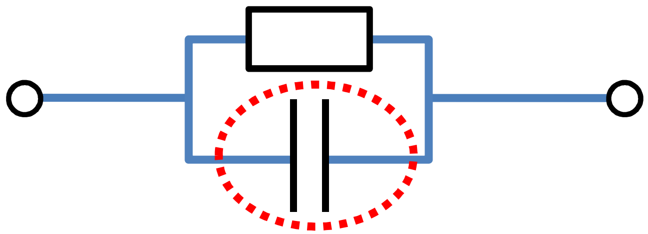
LCRZ Meters
Measure the Frequency-Dependent Impedance, with swept DC Bias.

Dependent on the settings,
this impedance is then converted into

either Resistor + Capacitor



or Resistor // Capacitor



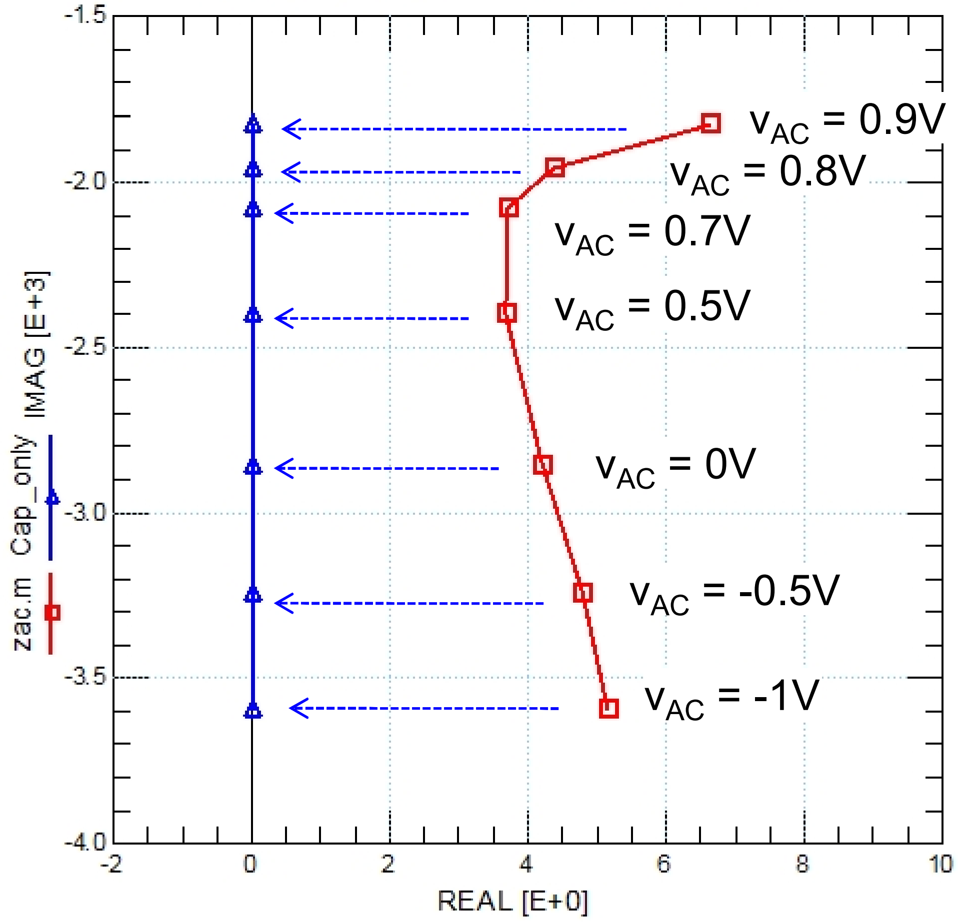
and, usually, **only the capacitance of the Resistor//Capacitor interpretation is applied to modeling**

The real world, however, is the measured, complex Impedance, while a CV measurement curve is just its projection to the y-axis

For Example:

Diode Impedance Measurement @ 1MHz

CV Measurement @ 1MHz



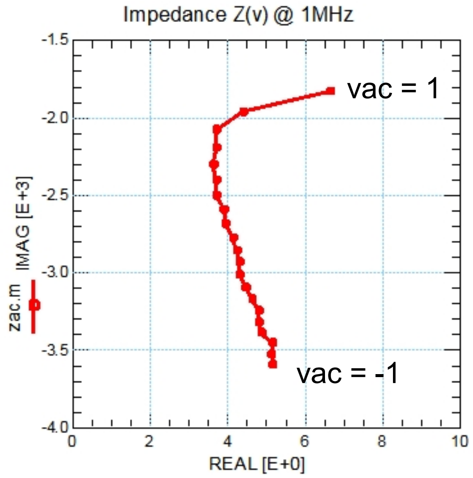
- ***all* physical capacitors also exhibit a loss, the *dissipation factor*.**
  This shows up like a resistor in series to a capacitor.
  In an Impedance Plot, this means a shift of the impedance curve to the right.

- when modeling *just the capacitor*,
  i.e. the projection of the reality to the y-axis,
  you will certainly get a fit,
  but the model may not be the correct, physical one.

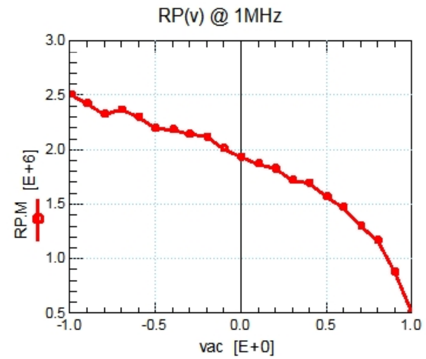
How to Read Capacitance and *Parallel Resistor* out of an Impedance Z Measurement:

```
|<---- Y --->|
      __
  --|__|--
  |   Rp  |
o-o---||---o-o
     Cp
```

$$Y = REAL(Y) + j \cdot IMAG(Y) = \frac{1}{Rp} + j \cdot \omega \cdot Cp$$

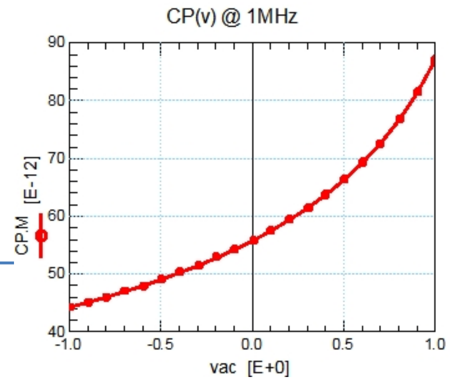


$$Rp = \frac{1}{REAL\left(\frac{1}{Z}\right)}$$



$$j \cdot IMAG(Y) = j \cdot \omega \cdot Cp$$

$$Cp = \frac{IMAG\left(\frac{1}{Z}\right)}{\omega}$$



And ...

How to Read Capacitance and *Series Resistor* (Dissipation Factor of Capacitor) out of an Impedance Z Measurement:

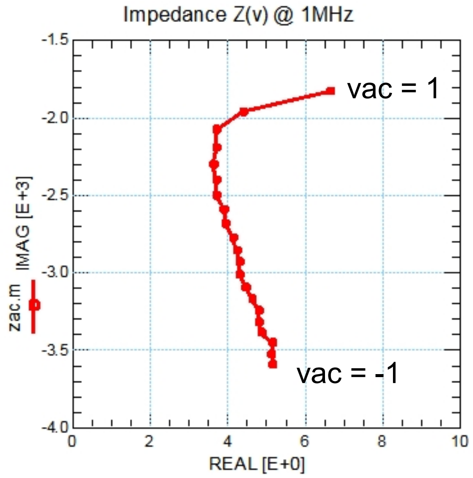
```
|<------ Z ------>|
        __
o---|__|--o--||---o
    Rs        Cs
```

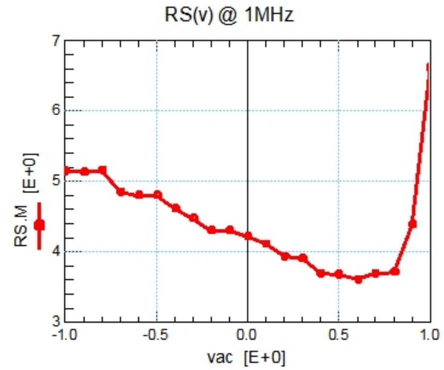
$$Z = REAL(Z) + j \cdot IMAG(Z) = Rs + \frac{1}{j \cdot \omega \cdot Cs}$$

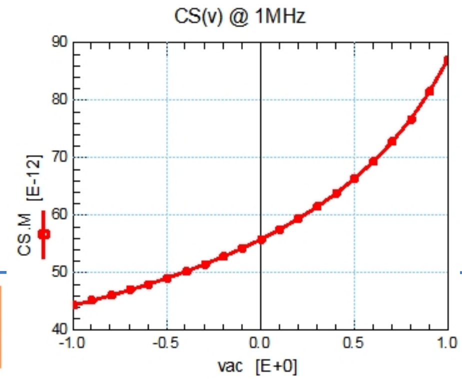
$$Rs = REAL(Z)$$

$$j \cdot IMAG(Z) = \frac{1}{j \cdot \omega \cdot Cs} = \frac{-j}{\omega \cdot Cs}$$

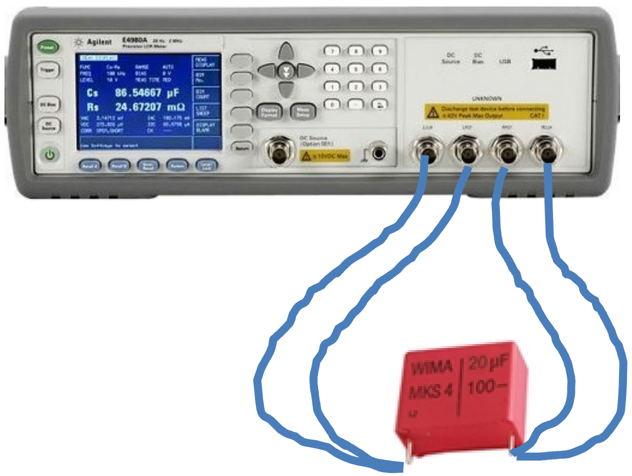
$$Cs = \frac{-1}{\omega \cdot IMAG(Z)}$$



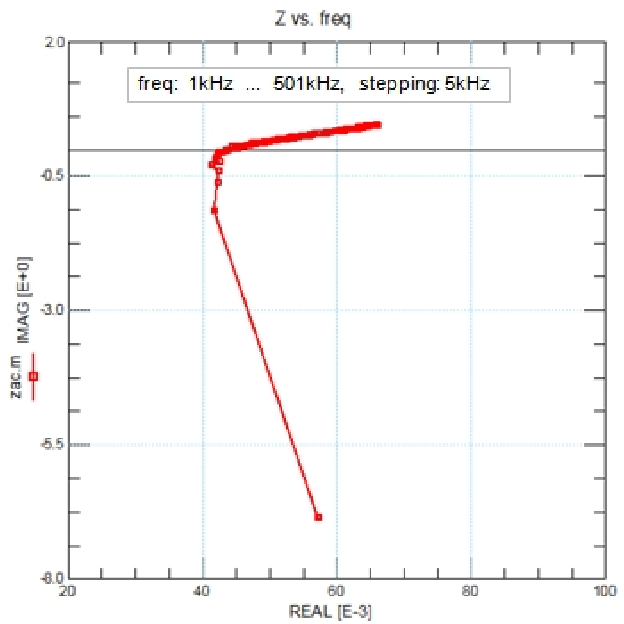


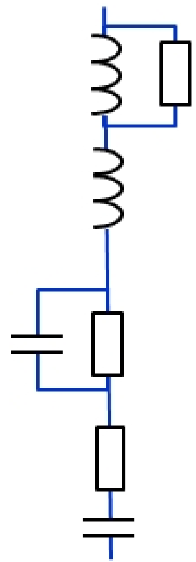


Practical Aspects of Impedance Analyzer Measurements



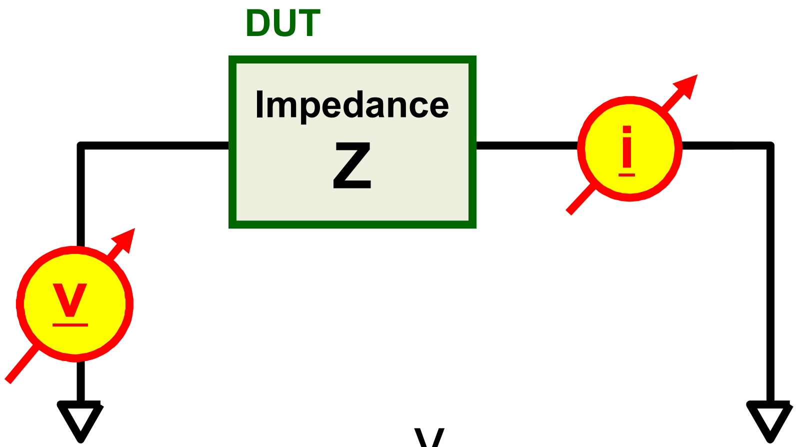
Impedance Plot 20µF Electrolyte Capacitor





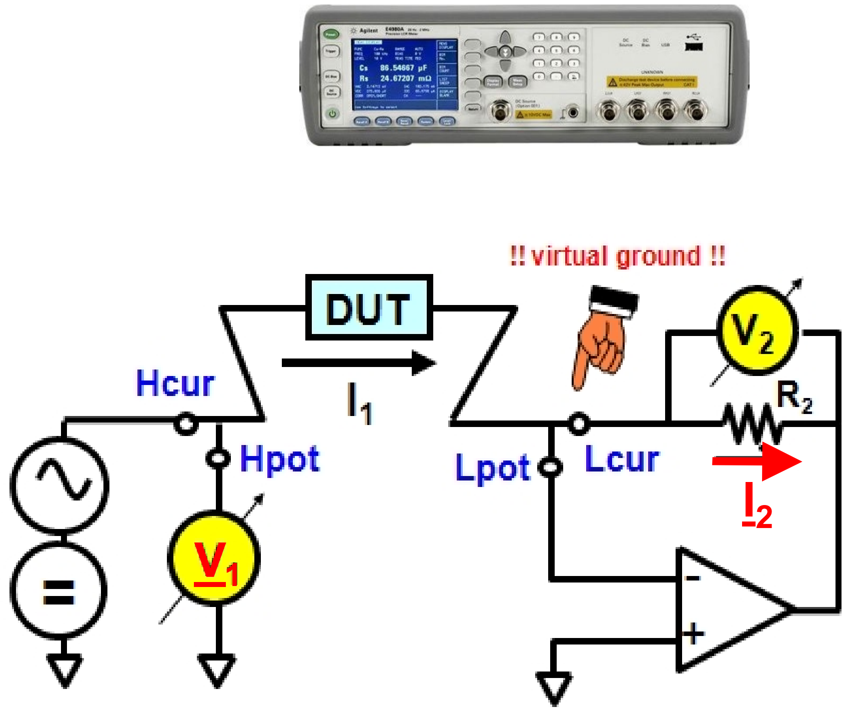
The Basic Impedance Analyzer Measurement Principle

"Diagonal Measurement"

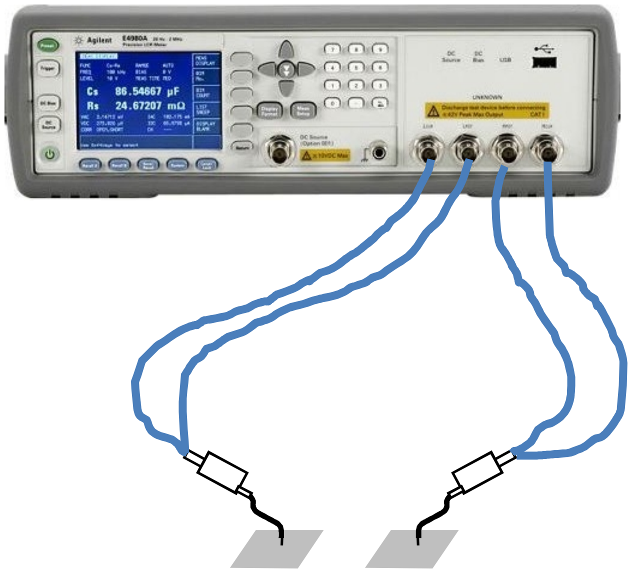


$$\underline{Z} = \frac{\underline{v}}{\underline{i}}$$

Auto Balancing Bridge Method



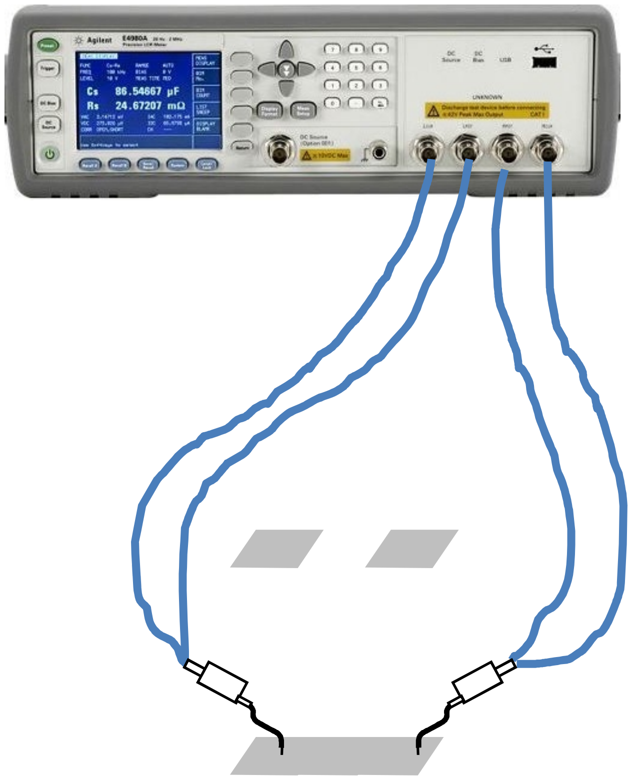
Impedance Analyzer Calibration



The impedances
of two Calibration Standards
are measured first

- **OPEN Cal. Standard measurement**

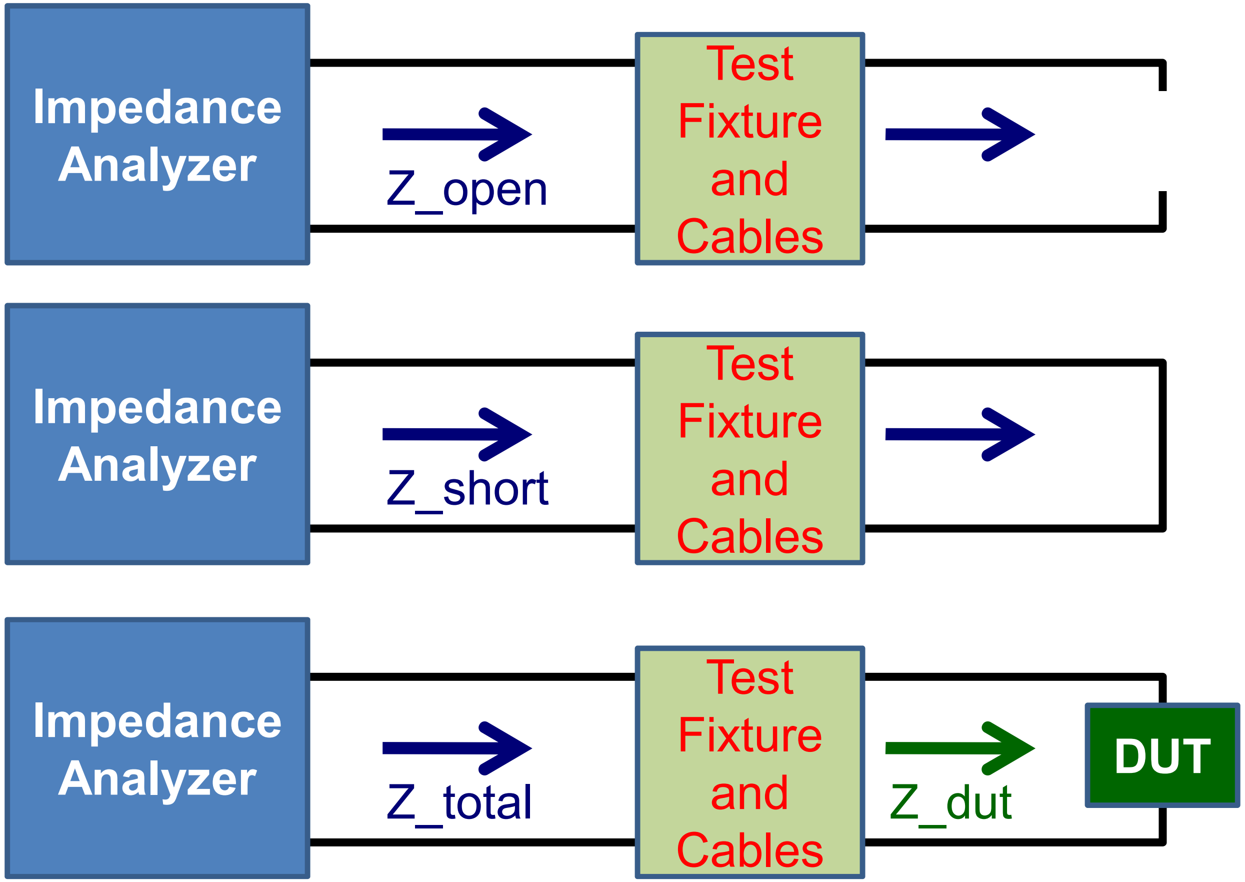
Impedance Analyzer Calibration



The impedances
of two Calibration Standards
are measured first

- **OPEN Cal. Standard measurement**

- **SHORT Cal. Standard measurement**



With Z_open and Z_short known, the DUT impedance can be calculated from Z_total

OPEN-only Calibration:

Z_dut = (Z_open*Z_total) / (Z_open - Z_total)

OPEN-SHORT Calibration:

Z_dut = (Z_short - Z_total) // (Z_total - Z_open) * Z_open

Pre-Requisite:
the equivalent schematic of test fixture and cables must be symmetrical.

In Practice:
not too long cables, good connectors

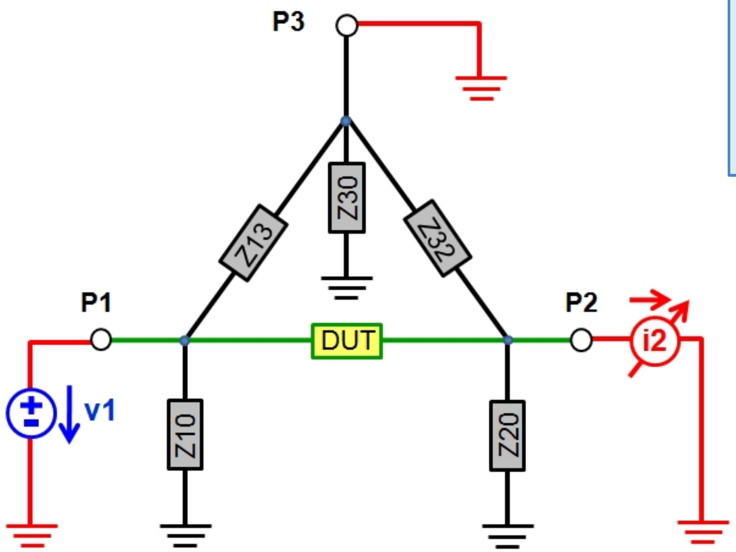
Source: Keysight Technologies, 'Impedance Measurement Handbook' 2014, Appendix B

Meas. and Simul. Principle of LCRZ Meters for ***Multi-Port Devices***:

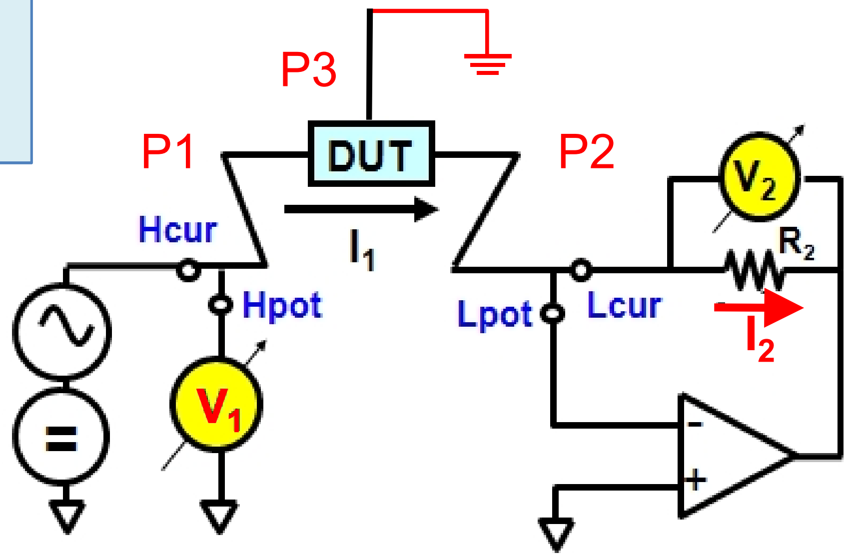
- stimulate voltage at one port
- measure the current at the other port
- connect not-involved nodes to ground

and as a result,

- **parasitics at each port to ground are not included in measurement result !**



$$\underline{Z}_{12} = \frac{\underline{v}_1}{\underline{i}_2}$$



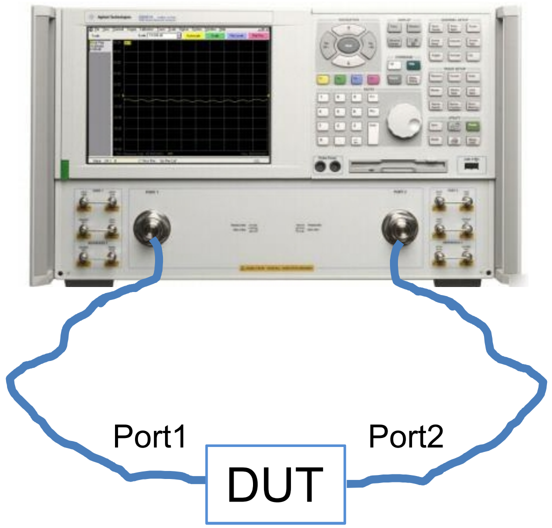
Outline

- The Impedance Plane Z = R + j*X
  and Typical Impedance Traces

- Impedance Plots from LCRZ Meters

- **Impedance Plots from S-Parameters**

Impedance Plots can also be obtained from S-Parameter Measurements



Calculating 1-Port S-Parameters from 2-Port:

Viewed from Port1, with Port2 *shorted*:

$$S\_1Port = S11 - \frac{S12 \cdot S21}{1 + S22}$$

Viewed from Port2, with Port1 *shorted*:

$$S\_1Port = S22 - \frac{S12 \cdot S21}{1 + S11}$$

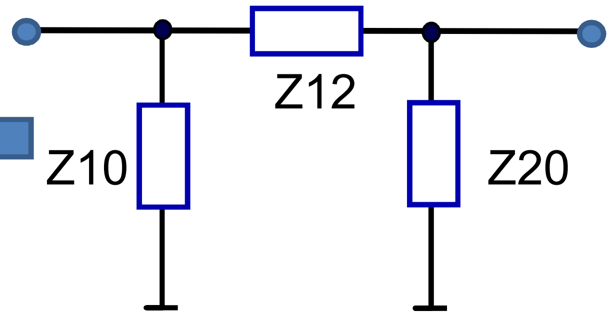
$$Impedance = \frac{1 + S\_1Port}{1 - S\_1Port} \cdot Z0$$

Interpreting Two-Port S-Parameter Measurements by a PI Schematic

$$\text{S matrix} = \begin{pmatrix} S.11 & S.12 \\ S.21 & S.22 \end{pmatrix}$$

Assuming an underlaying PI schematic for the DUT, convert the de-embedded S-parameters to Y-parameters, and calculate the impedances of the PI schematic branches

$$\text{Y matrix} = \begin{pmatrix} Y.11 & Y.12 \\ Y.21 & Y.22 \end{pmatrix} = \begin{pmatrix} \frac{1}{Z10} + \frac{1}{Z12} & \frac{-1}{Z12} \\ \frac{-1}{Z12} & \frac{1}{Z20} + \frac{1}{Z12} \end{pmatrix}$$



$$\frac{1}{Y.11 + Y.12} = Z10$$

$$-\frac{1}{Y.12} = Z12$$

$$\frac{1}{Y.22 + Y.21} = Z20$$

all variables are represented by complex numbers

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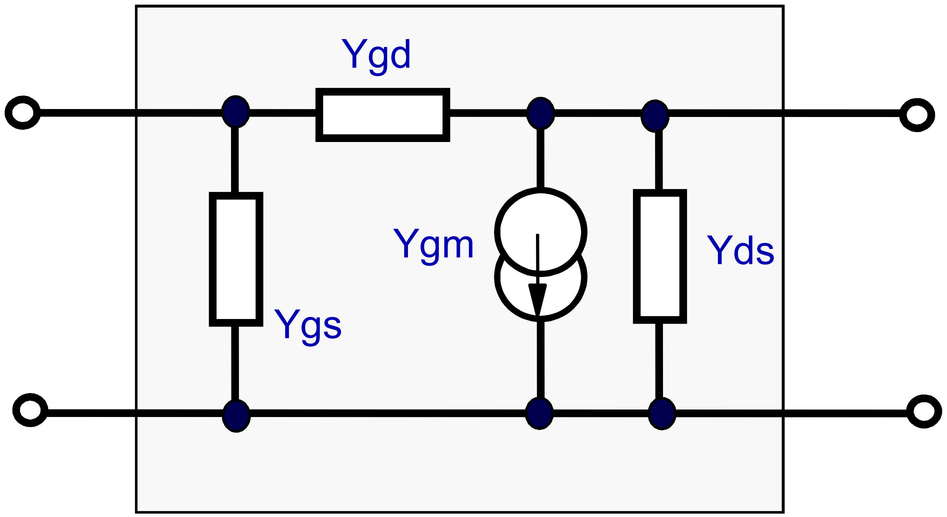
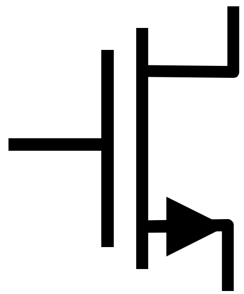
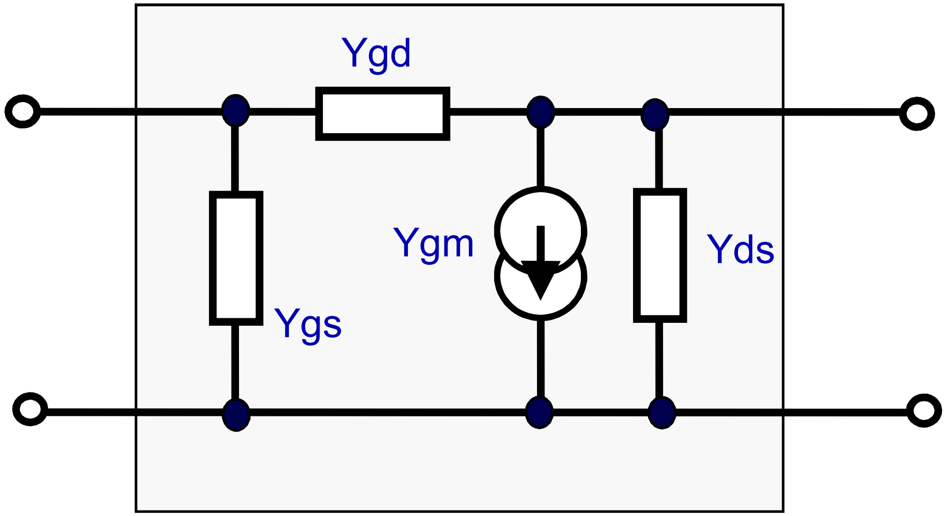
A Special Case:
Transistor PI Schematic Modeling





The Idea

- **Convert the S-Parameter Matrix**
  - **to a Y Matrix,**
  - **and apply the PI Schematic Interpretation for Transistor Modeling**

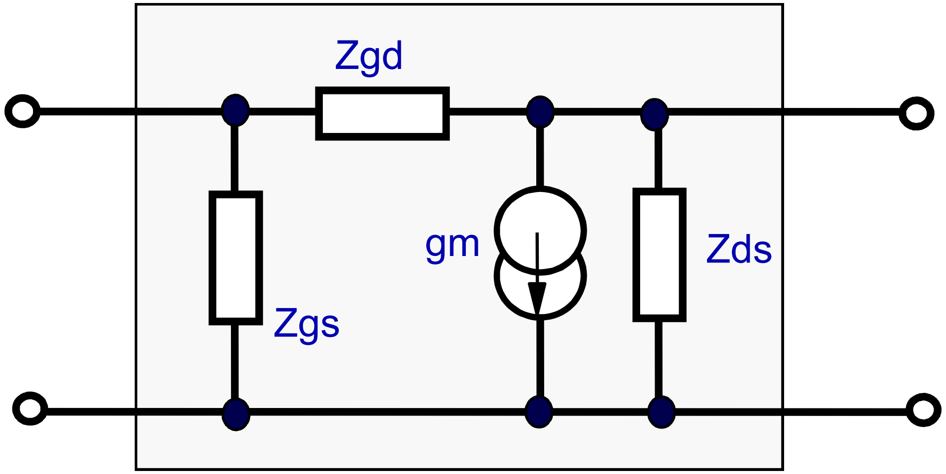


$$\begin{pmatrix} Y11 & Y12 \\ Y21 & Y22 \end{pmatrix} = \begin{pmatrix} Ygs + Ygd & -Ygd \\ Ygm - Ygd & Yds + Ygd \end{pmatrix}$$

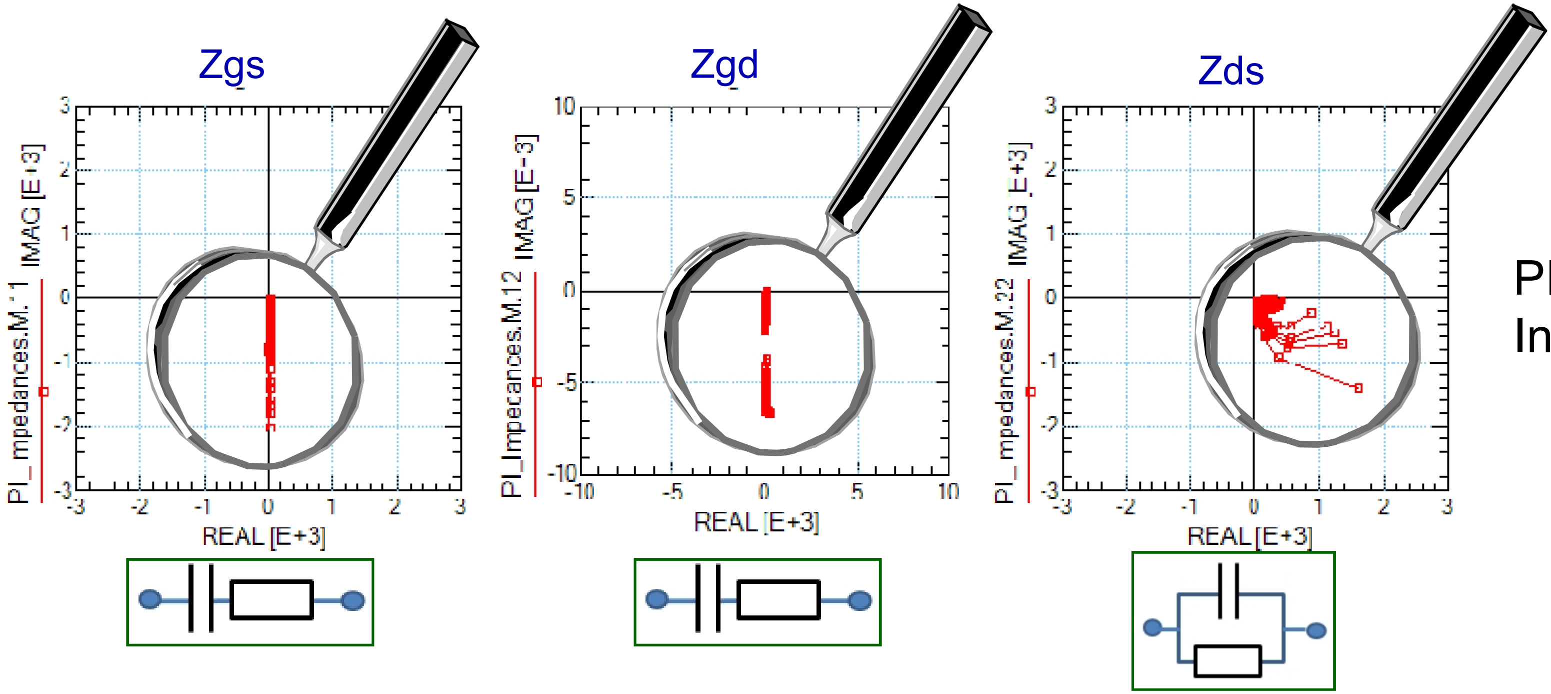
Ygs = Y11 + Y12
Ygd = -Y12
Ygm = Y21 - Y12 = gm * exp(-jωTAU)
Yds = Y22 + Y12

A Best-Practice Intermediate Step:

Inspect/Verify First the PI-Schematic Impedances

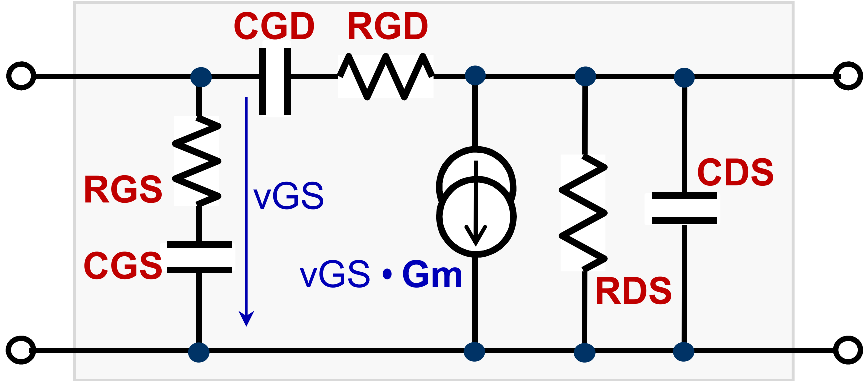


$Zgs = 1 / Ygs$
$Zgd = 1 / Ygd$
$Zds = 1 / Yds$



PI Impedances Inspection

How to Get the Inner PI Components for Quasistatic HEMT or MOSFET



1. Convert de-embedded S-parameters to Z, and strip-off external inductors and resistors

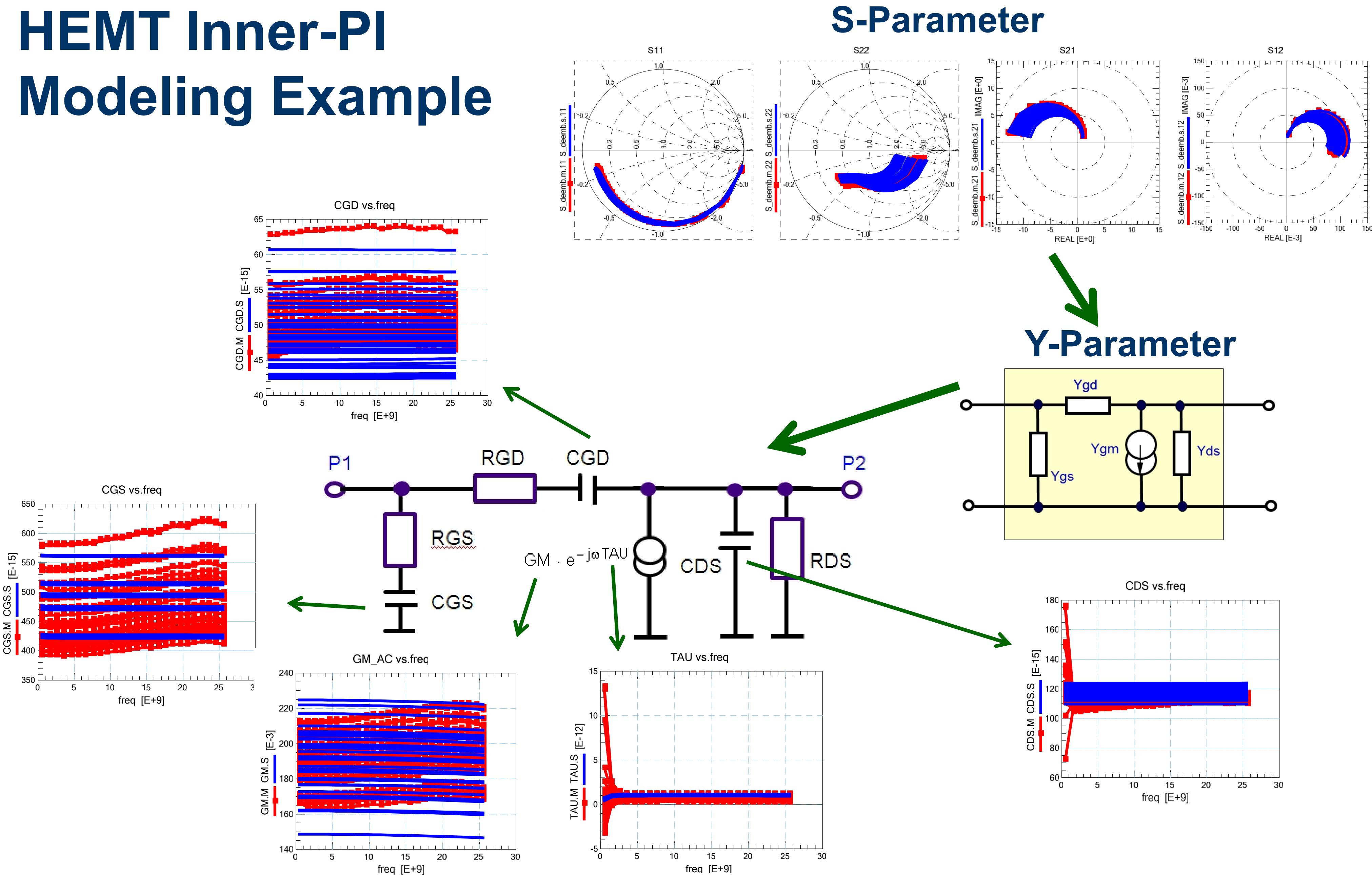
2. Convert to Y-parameters and calculate complex impedances, admittances and Gm

Z_10 = $(Y.11 + Y.12)^{-1}$	Impedance Port1 -> GND
Z_12 = $(-Y.12)^{-1}$	Impedance Port1 -> Port2
Gm = Y.21 - Y.12 = GM • $e^{-j \cdot 2PI \cdot freq \cdot TAU}$	Voltage -> Current Amplification
Y_20 = Y.22 + Y.12	Admittance Port2 -> GND

3. Finally, get

RGS = REAL(Z_10)	CGS = - $(IMAG(Z\_10)^{-1})$ / (2PI • freq)
RGD = REAL(Z_12)	CGD = - $(IMAG(Z\_12)^{-1})$ / (2PI • freq)
GM = MAG(Gm)	TAU = - PHASE(Gm) / (2PI • freq)
RDS = $(REAL(Y\_20))^{-1}$	CDS = IMAG(Y_20) / (2PI • freq)

HEMT Inner-PI Modeling Example



Calculating the Branch-to-Branch Impedances of *Multi-Ports*

- **The Y-matrix relates the currents into the ports with the stimulating port voltages.**
- **The matrix elements unit is admittance.**

measured | stimulus

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} * \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

The Y-matrix is very useful when the impedances between the ports need to be extracted and analyzed, especially for multi-port applications. This is due to the voltage stimulation at the ports.

☞ When interested in the impedance between a certain port A, and another port B, all voltages, except the one at port A , have to be set to zero. ***Then, the current for the impedance calculation is not measured at this port A, but rather at port B.***

Of course, all other shorted ports do also sink currents, provided by the voltage source at port A, but they are *not* involved in the port B current measurement.

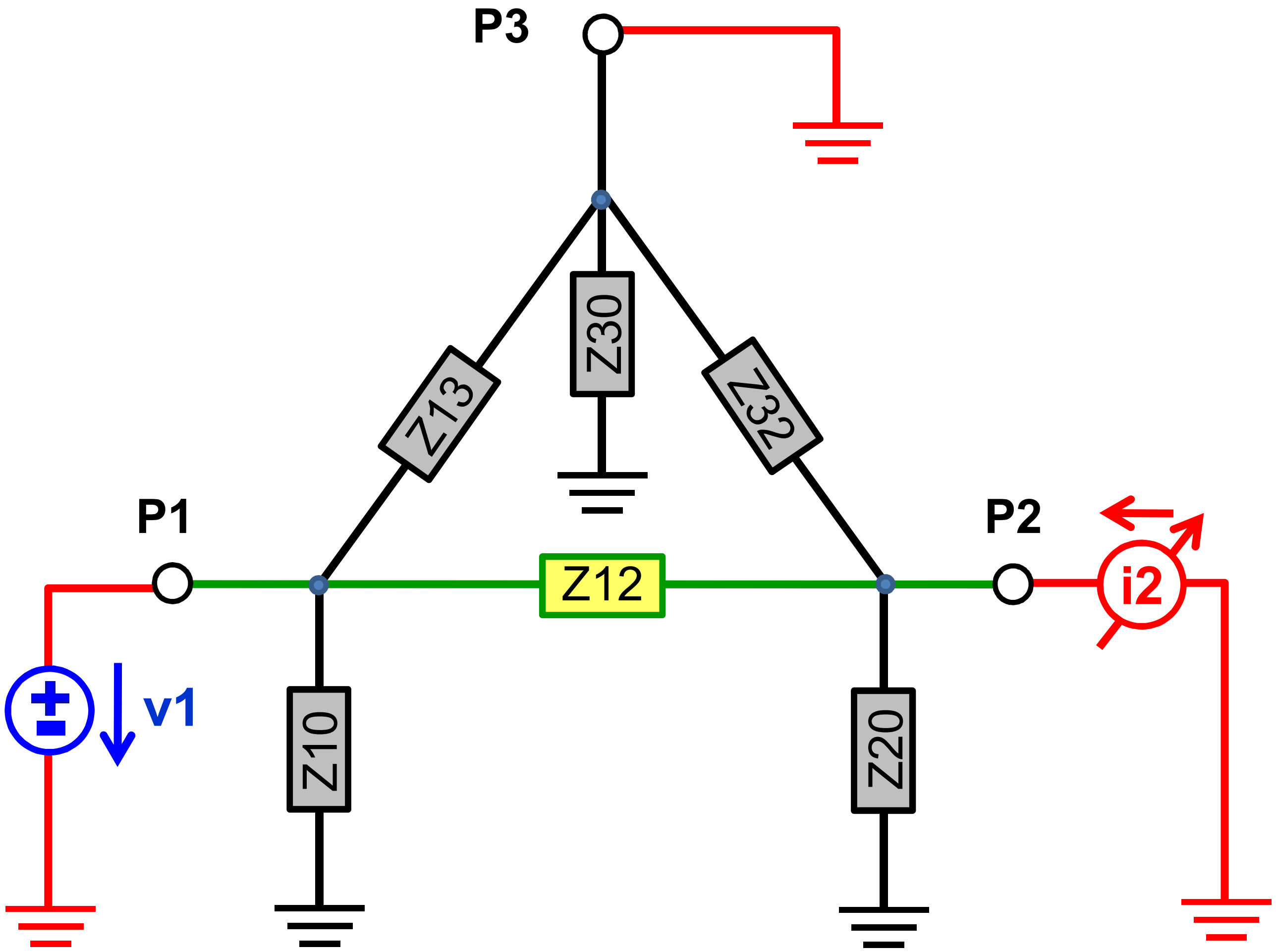
☞ **In other words, the impedance Z, between port A and the shorted port B, is simply**

$$Z_{BA} = -\frac{1}{Y_{BA}}$$

How to Calculate the Branch Impedances of a 3 Port

Example:

Port1-to-Port2-Impedance Z12



From an inspection of
the 3-Port Y-Matrix definition:

$$\begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{pmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

apply a SHORT to Port2 and Port3,
stimulate a voltage at Port1,
measure the current at Port2
and calculate:

$$Z12 = \frac{v1}{-i2} = -(Y21)^{-1}$$

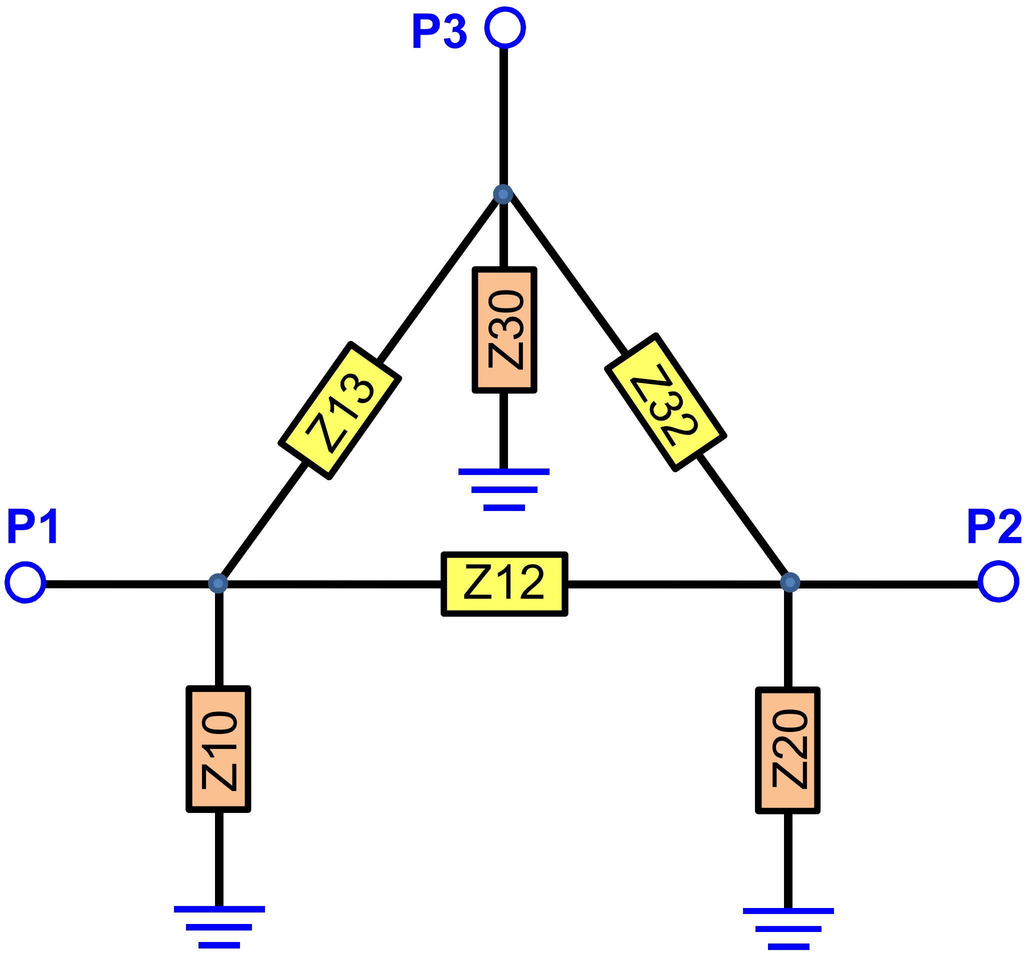
Note:
the Y-Matrix indexing is
Admittance$_{\text{to from}}$

e.g. the admittance
from Port1 to Port2 is Y21

Note: for the Y-matrix, mind the currents *into* the DUT

How to Calculate the Branch Impedances of a 3 Port

At a Glance:



From the 3-Port Y-Matrix:

$$\begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{pmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

calculate the inter-port branch impedances:

$$Z12 = -(Y_{12})^{-1}$$
$$Z13 = -(Y_{13})^{-1}$$
$$Z32 = -(Y_{32})^{-1}$$

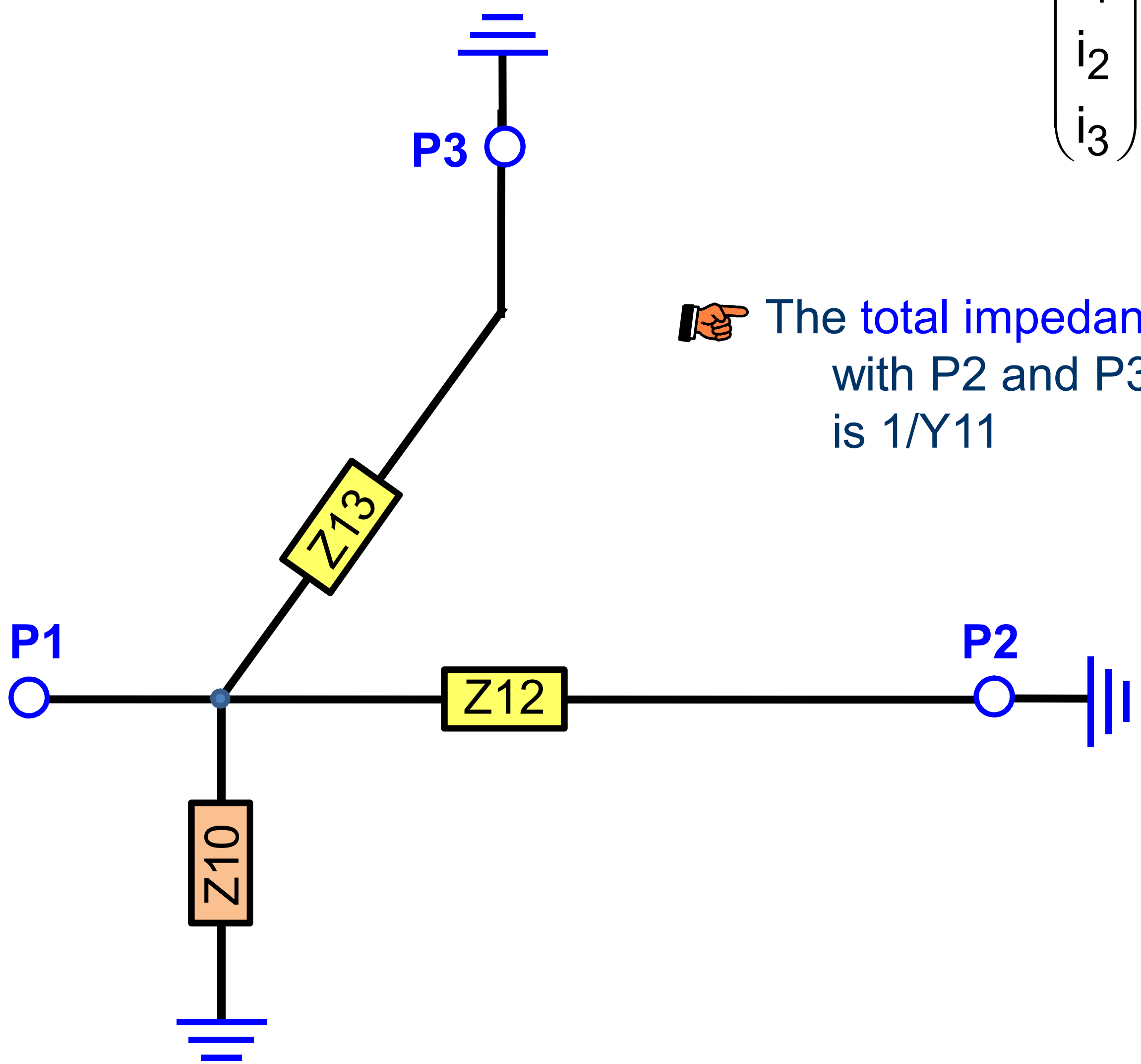
and the pin-to-ground impedances:

$$Z10 = (Y_{11} + Y_{12} + Y_{13})^{-1}$$
$$Z20 = (Y_{21} + Y_{22} + Y_{23})^{-1}$$
$$Z30 = (Y_{31} + Y_{32} + Y_{33})^{-1}$$

How to Calculate the Branch Impedances of a 3 Port

continued:

$$\begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{pmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

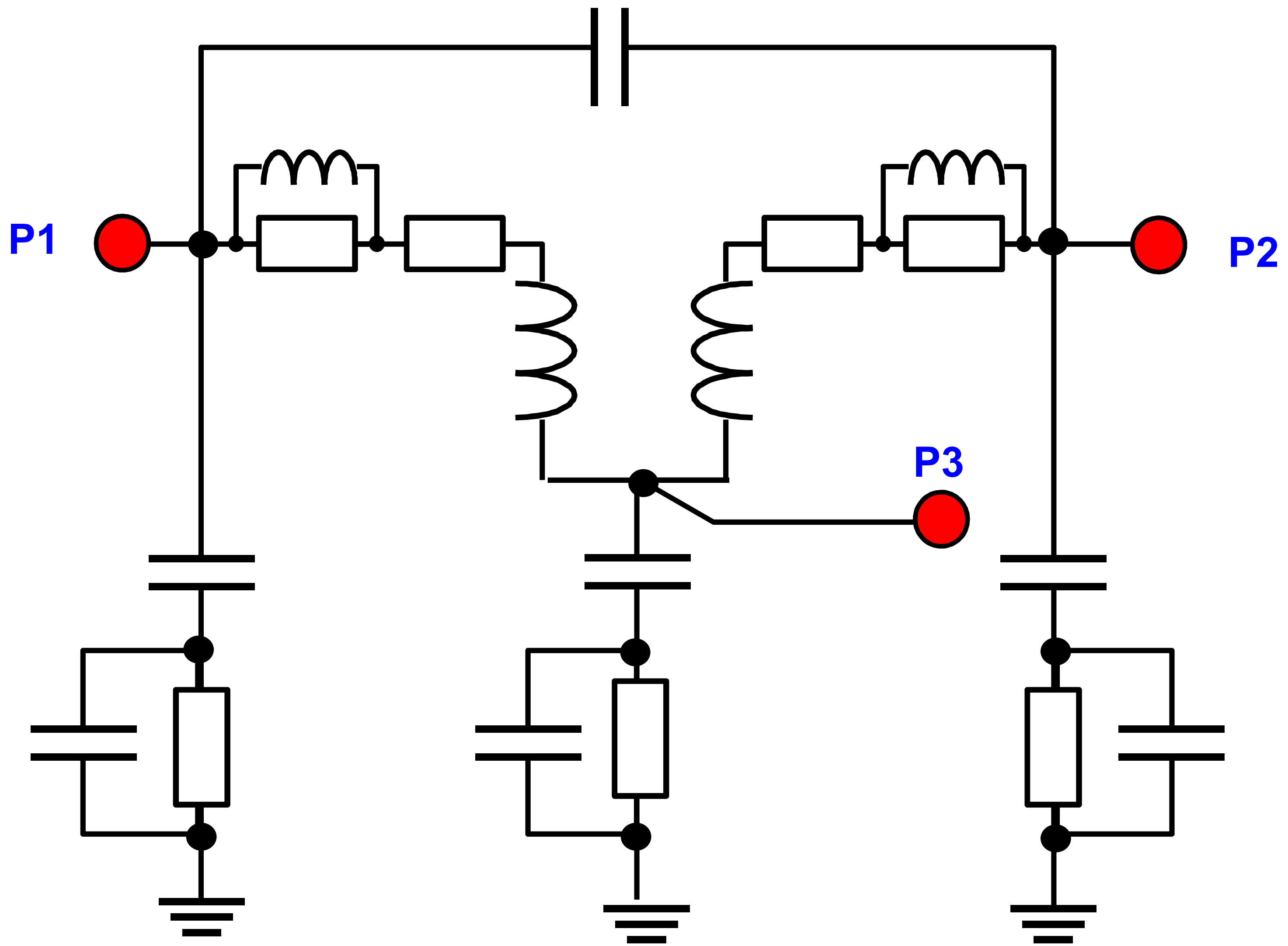


The total impedance at P1,
with P2 and P3 grounded,
is 1/Y11

and so on:
At P2, with P1 and P3 grounded,
the total impedance is 1/Y22

And at P3, with P1 and P2 grounded,
it is 1/Y33

Application Example: 3-Port Transformer



Wrap-Up

- The Impedance Plane Z = R + j*X
  and its interpretation is an important tool
  for device modeling engineers to develop
  accurate Spice models.

- Impedance Plots can be obtained by LCRZ Meters

- and from S-Parameters of Network Analyzers

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