APPLICATION NOTE

Curve Fitting for Spice Device Modeling

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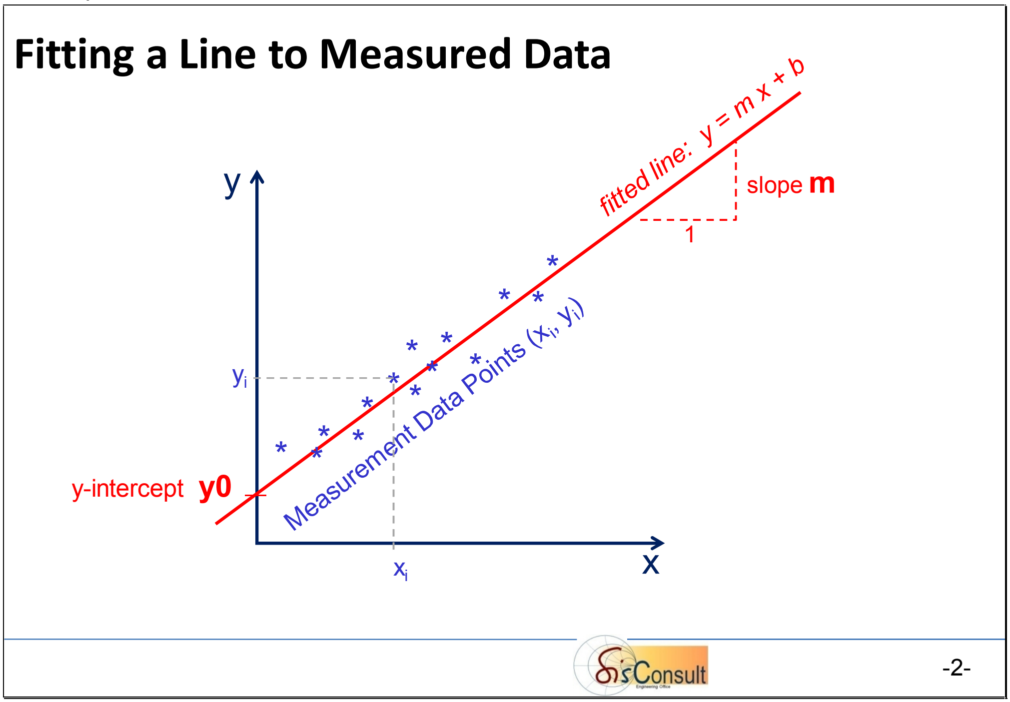
Curve Fitting Applying Regression Analysis

Fitting a line to measured data is a common task for modeling engineers.
The fundamental tool is the so-called 'Linear Regression Analysis'.
This means to fit a straight line, represented by equation $y = y0 + m*x$, in the best way to (a range of) measured data, as depicted below:



The formulas given below explain how to calculate the slope and the y-intercept of the fitted line from the measured data (x_i/y_i).

The Linear Regression Analysis Formulas

Slope

$$m = \frac{\sum_{i=1}^{N} x_i \cdot \sum_{i=1}^{N} y_i - N \cdot \sum_{i=1}^{N} x_i \cdot y_i}{\left(\sum_{i=1}^{N} x_i\right)^2 - N \cdot \sum_{i=1}^{N} x_i^2}$$

y-Intercept

$$y0 = \frac{1}{N} \cdot \left(\sum_{i=1}^{N} y_i - m \cdot \sum_{i=1}^{N} x_i\right)$$

Regression Coefficient
(a measure of fitting quality)

$$r^2 = m^2 \cdot \frac{\sum_{i=1}^{N} x_i^2 - \frac{1}{N} \cdot \left(\sum_{i=1}^{N} x_i\right)^2}{\sum_{i=1}^{N} y_i^2 - \frac{1}{N} \cdot \left(\sum_{i=1}^{N} y_i\right)^2}$$

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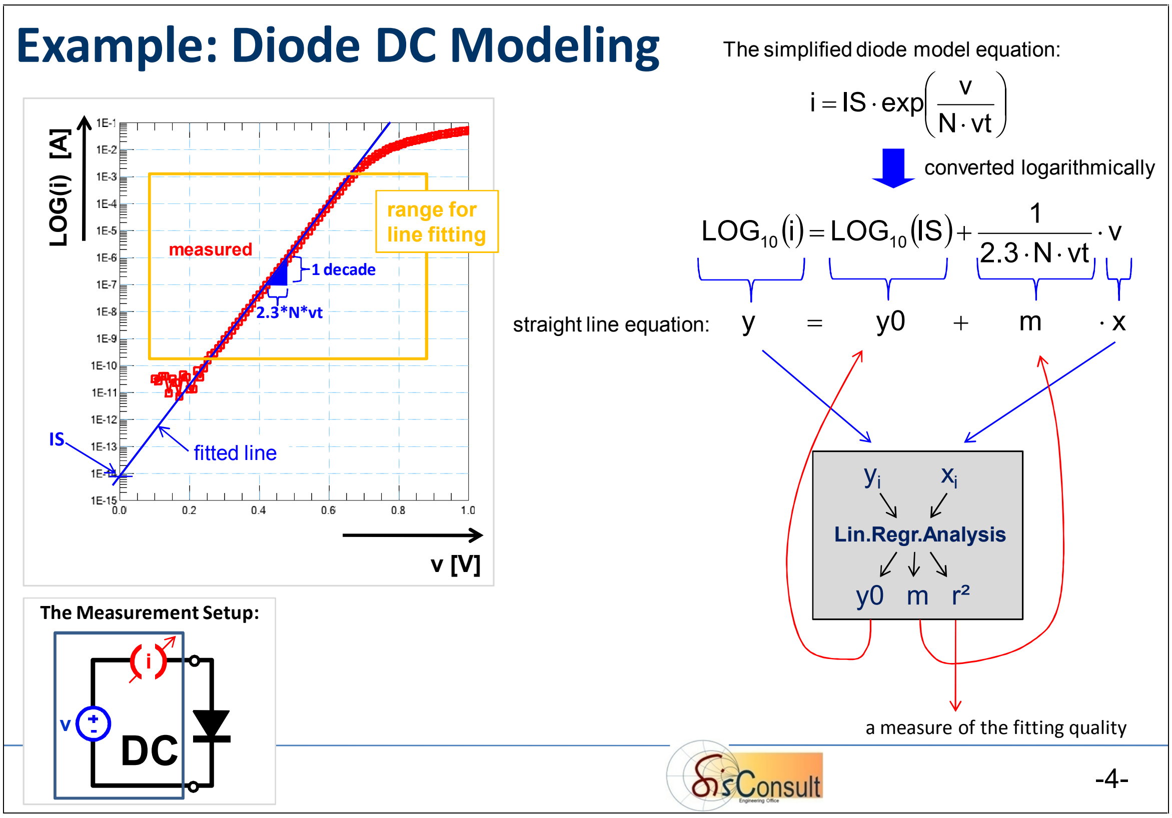
Also given is the formula for the 'Regression Coefficient' r^2.
This coefficient is a measure for the fitting quality:

- if $r^2=1$, then all measured data would lie exactly on the fitted line.
- If $r^2=0$, then the measured data represented a cloud: no line can be fitted.

Typical values of r^2 for typical measurement data are in the range [0.95 ...0.99].



Shown above is a quick example of regression analysis: the modeling of the DC current of a diode in forward bias. This is at the same time also an example of how the simple linear regression formulas of the slide before can be applied also to non-linear fitting problems.

The diode model equation is

$$i = IS * \left(\exp\left(\frac{v}{N * vt} \right) - 1 \right) \tag{1}$$

Provided v>0, i.e. neglecting the term (-1) in (1), and applying a LOG_{10} conversion, gives:

$$LOG_{10}(i) = LOG_{10}(IS) + \frac{v}{N * vt} LOG_{10}(e)$$

or

$$LOG_{10}(i) = LOG_{10}(IS) + \frac{1}{2.3 * N * vt} * v \tag{2a}$$

This can be interpreted as a linear equation of the form:

$$y = y0 + m * x \tag{2b}$$

what means, we have to substitute:

$y = LOG_{10}(i)$ (3a)

$y0 = LOG_{10}(IS)$ (3b)

$m = [1 / (2.3\ N\ vt)]$ (3c)

$x = v$ (3d)

This substitution corresponds to the required manipulations for the measured data:
after the logarithmic conversion of the measured values of the diode current 'i' following equ.(3a),

they are introduced together with the unmanipulated values of the diode voltage 'v' (see equ. 3d) into the regression equations of the previous slide, as y_i- and x_i-values.
The regression formulas return the y-intercept 'y0' and the slope 'm' of the fitted line.

To finally obtain the Spice parameter values, we have to solve equ. (3b) for the diode model parameter 'IS' and equ. (3c) for 'N', and obtain:

$$IS = 10^{y0}$$

and

$$N = 1 / (2.3\ m\ vt)$$

with the temperature voltage

```
vt = 8.6171E-5 * (TEMP['C] + 273.15)
```

what are the Spice model parameters for the blue fitted line in the slide above.

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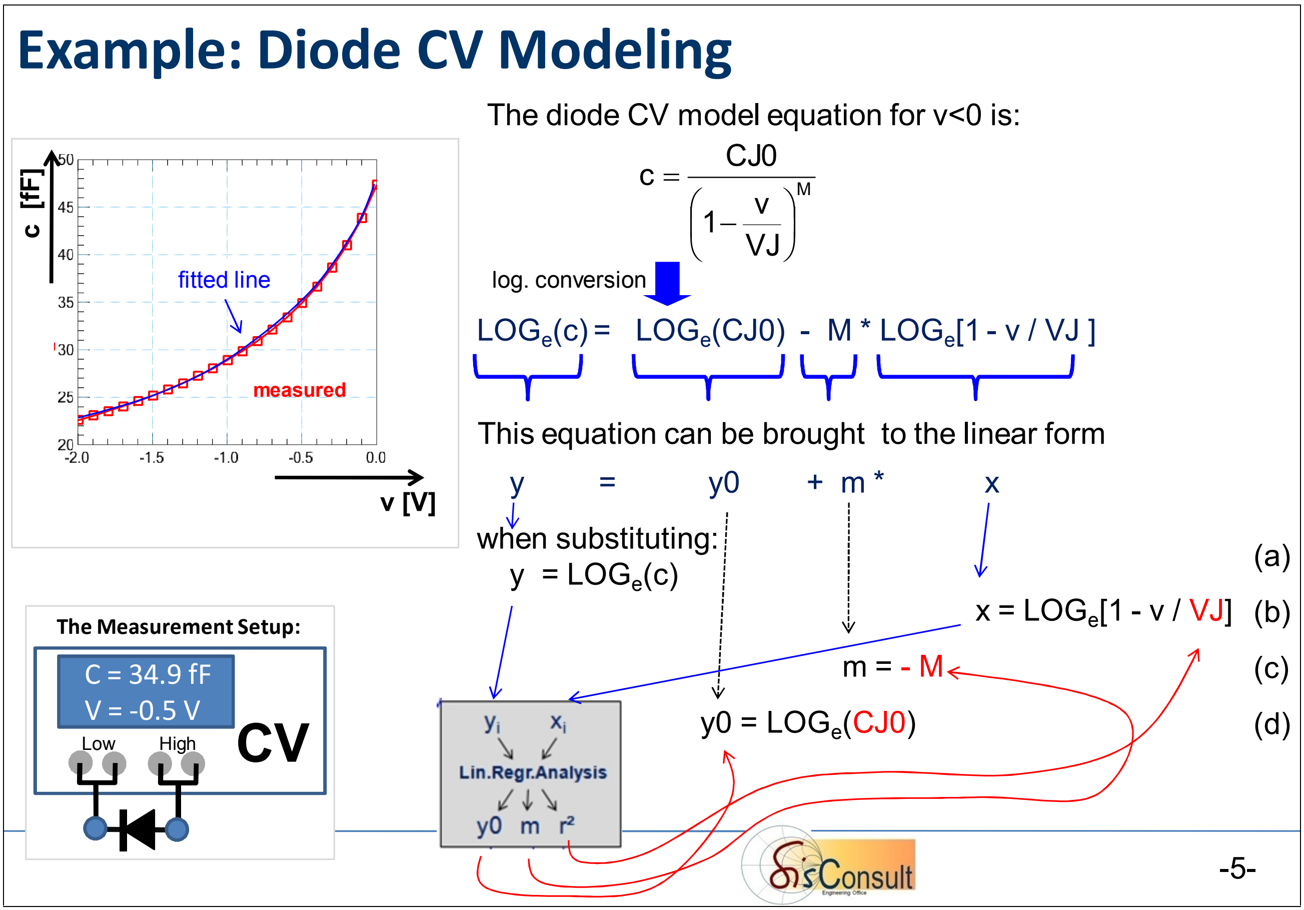

Although the target formula for the voltage-dependent diode capacitance is even hyperbolic in this example, we once again apply the linear regression fitting:

- the stimulus voltage values v[i] are converted following equation (b),
- and the measured capacitance values Cs[i] following equ. (a).

The problem of fitting three unknown model parameters ('CJ0', 'VJ' and 'M') to a linear equation with only 2 parameters ('y0' and 'm') is resolved by knowing that the parameter VJ lies usually between ~0.3 < VJ < ~1, and by considering the value of the regression analysis fitting coefficient r^2 (!).

Beginning with a starting value for 'VJ', e.g.. 0.3V, we populate the two data arrays x[i] and y[i] with the measured data, following equations (a) and (b), and introduce these arrays into the linear regression formulas. Then, we obtain a slope value m(VJ) and an y-intercept y0(VJ) for the current value of VJ. As mentioned, we also regard the fitting coefficient r^2. All three values of m, y0 and r^2 are stored.

Next, VJ is incremented to e.g. 0.31V. Then, the same data transform is applied once again to the measurement data v[i] and c[i] , and another regression analysis is performed.
From that we get another triplet of coefficients m[VJ], y0[VJ], and r^2[VJ].

This process is iterated until the best fitting coefficient r^2 is obtained, corresponding to VJ_opt, m(VJ_opt) and y0(VJ_opt).
A final re-substitution gives:
from (c): M = - m[VJ_opt]
from (d): CJ = exp (y0[VJ_opt])

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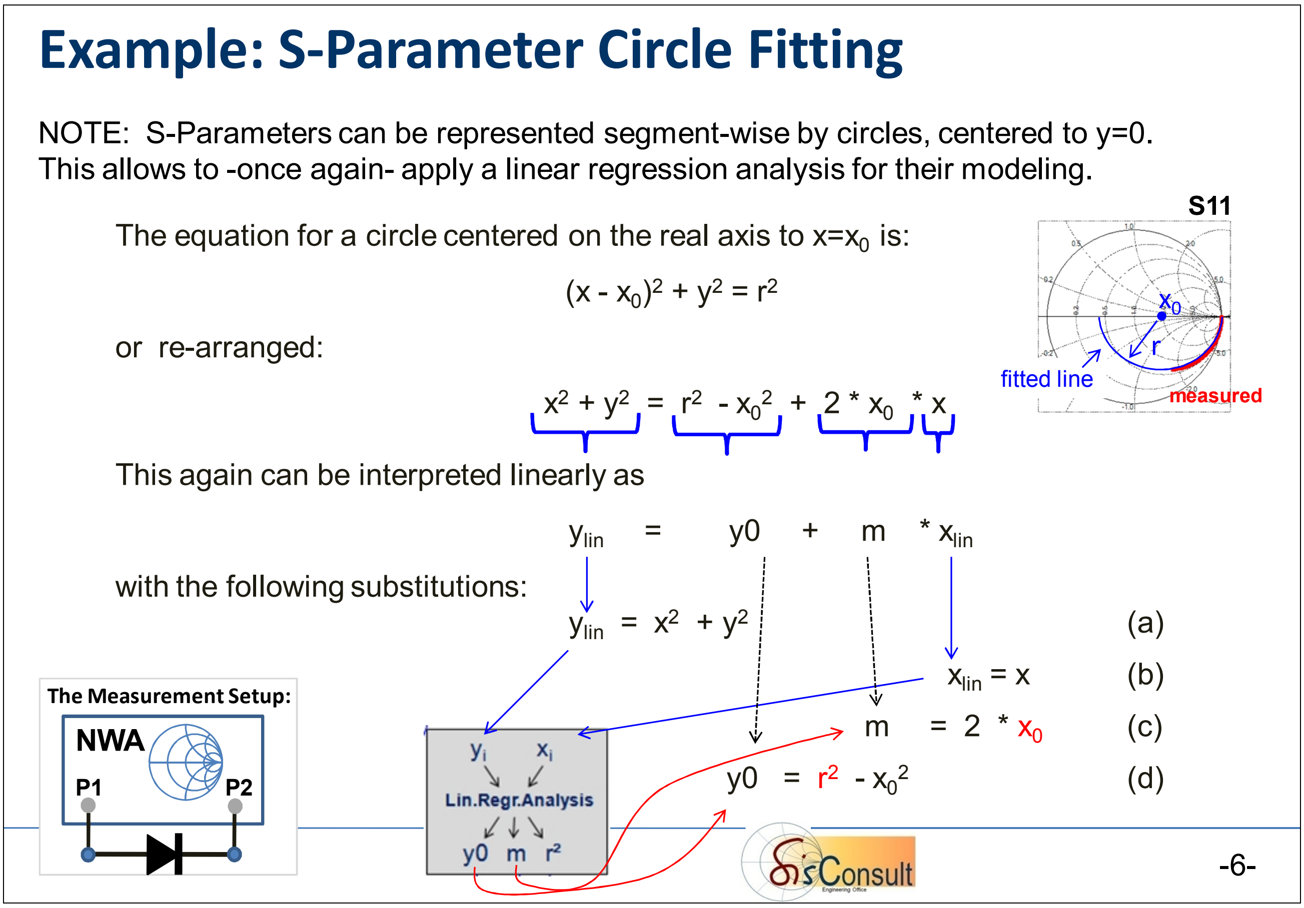

This looks like a very exciting example: fitting a half-circle by applying linear regression !

How to proceed:

from the measured S-parameter values x[i] =REAL(Sxx[i]) and y[i]=IMAG(Sxx[i]), we get the y_{lin} values after equation (a), and the values x[i]=REAL(Sxx[i]) become xlin values after (b).

Again, a linear regression fit is performed.

From the slope 'm' and y-intercept 'y0' of the fitted line, we obtain the circle specifics

- location of circle center on the x-axis:

  $x_0 = m / 2$ from equation (c)

- and circle radius:

  $r = SQRT(y0+x_0^2)$ from equation (d)

See the blue fitted line in the slide above.

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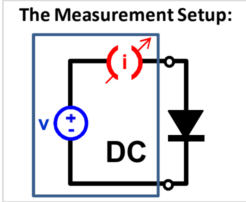
Visual Model Parameter Extraction

- Converting the Measurement Result into Parameter Plots -

The Basic Idea

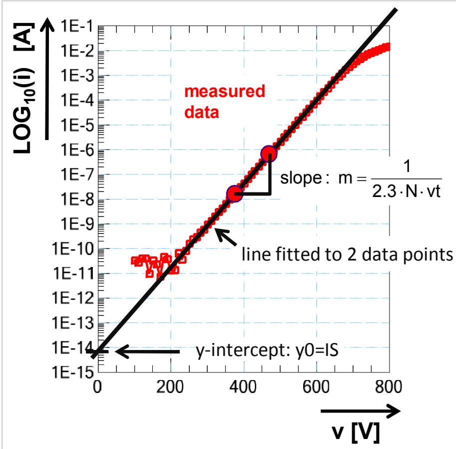


Example: Diode

With the assumption of v>0.1V, the diode model equation is $i = IS \cdot \exp\left(\frac{v}{N \cdot vt}\right)$

When the measured DC data are plotted semi-logarithmically, i.e. 'LOG_{10}(i)' vs. 'v', as on the right, the y-intercept of a fitted line represents the model parameter 'IS', while the parameter 'N' is represented by the slope, as depicted.

By plotting the y-intercepts and the slopes of all measured data points (x_i/y_i) versus -again- the stimulus voltage 'v', the valid parameter extraction range is represented by constant parameter values, as shown in the next slide.



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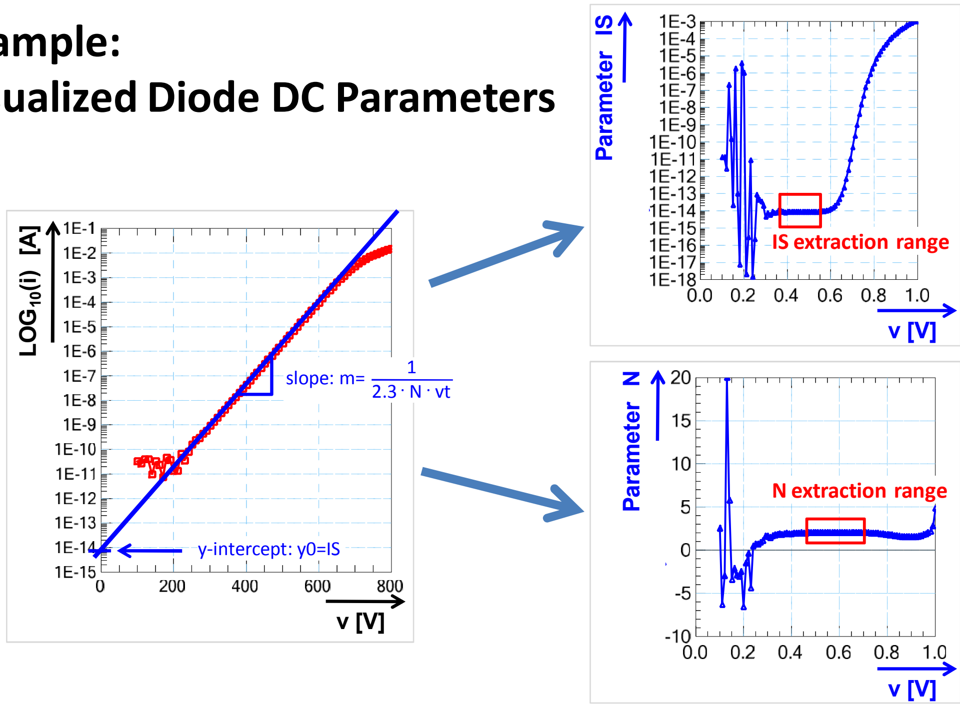
Example: Visualized Diode DC Parameters



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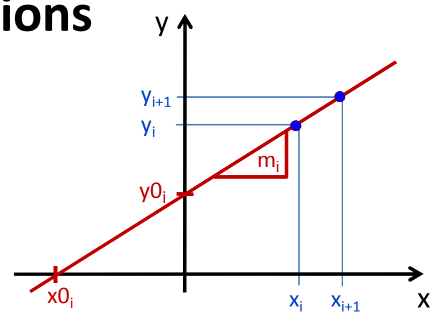
For the example of a diode the above slide shows the result of visualizing the DC parameters 'IS' and 'N'. The big advantage of this modeling method is a clear indication *where* the model and its parameters are valid for being extracted (applying a mean value from the red boxes depicted in the slide above).
In other words: the Spice model parameter extraction itself is a simple mean value calculation from the parameter values within the flat range, as sketched above.

How to perform the parameter visualization:

Performing visual parameter extraction only requires a program loop across all measured data, providing the slope, the x- and the y-intercept as a function of the stimulus 'x'.

The Three Fundamental Equations



The equation for a straight line is:

$$y = y0 + m \cdot x$$

When fitting this line to two data points (x_i / y_i), (x_{i+1} / y_{i+1}), its slope is:

$$m_i = \frac{y_{i+1} - y_i}{x_{i+1} - x_i}$$

its x-intercept $x0_i$ is:

$$x0_i = \frac{x_i \cdot y_{i+1} - x_{i+1} \cdot y_i}{y_{i+1} - y_i}$$

and the y-intercept $y0_i$:

$$y0_i = \frac{x_{i+1} \cdot y_i - x_i \cdot y_{i+1}}{x_{i+1} - x_i}$$

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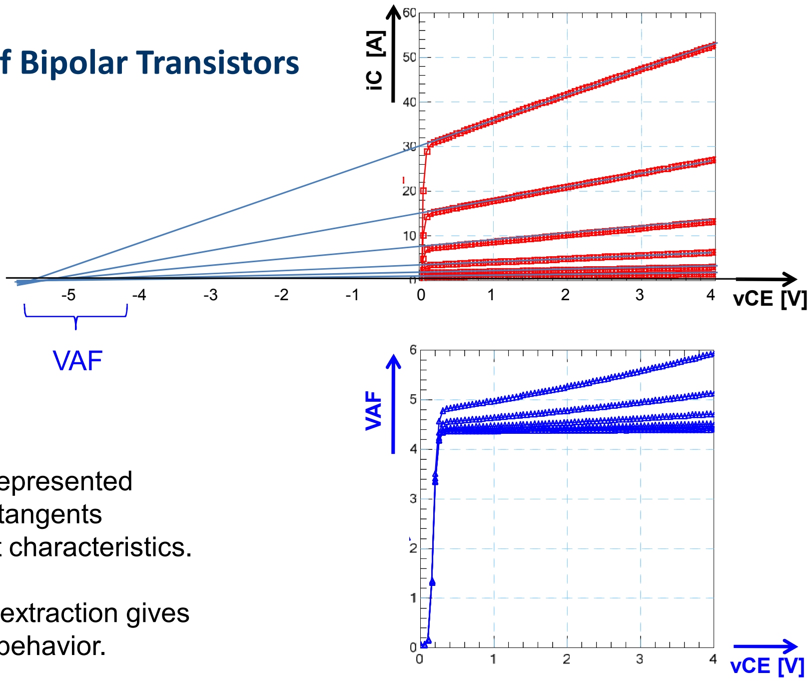
Example: Early Voltage VAF of Bipolar Transistors

Example:
Early Voltage VAF of Bipolar Transistors



The Early Voltage is represented by the x-intercepts of tangents fitted to the DC output characteristics.

The visual parameter extraction gives a clear picture of this behavior.

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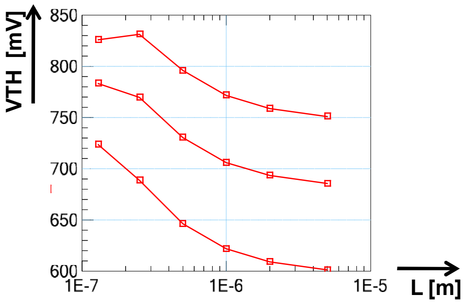
Example: MOS Transistor VTH vs. L or W

Applying Visual Parameter Extraction for Model Development



The threshold voltage 'VTH' of MOS transistors with different device sizes, plotted versus length 'L' or width 'W', shows a characteristic trace.

In the first MOS models of the 1970ies, the threshold voltage modeling was done by a single parameter 'VTH'. In other words, no dependency of the threshold voltage vs. geometry was included.

Applying the visual parameter extraction method assisted with improving the MOS model by replacing (in the model equations!) the single model parameter VTH by a function VTH = f(L, W), as given by the visualized parameter plots.

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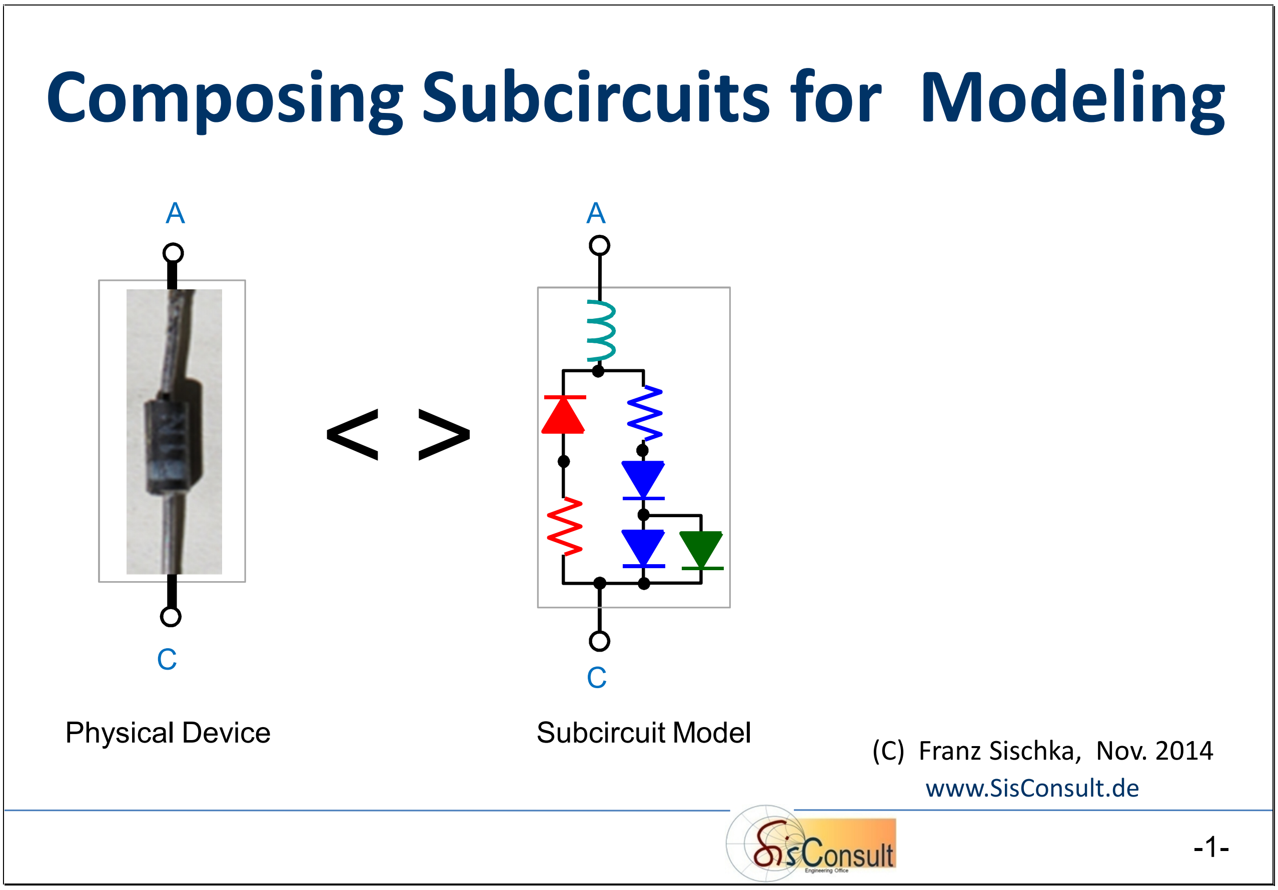
Composing Spice Subcircuits for Modeling



When modeling electronic devices, the first choice is to select a model which describes best the behavior of the physical device.

If, however, the model does not cover the measurements accurately enough, the modeling engineer has the choice of either

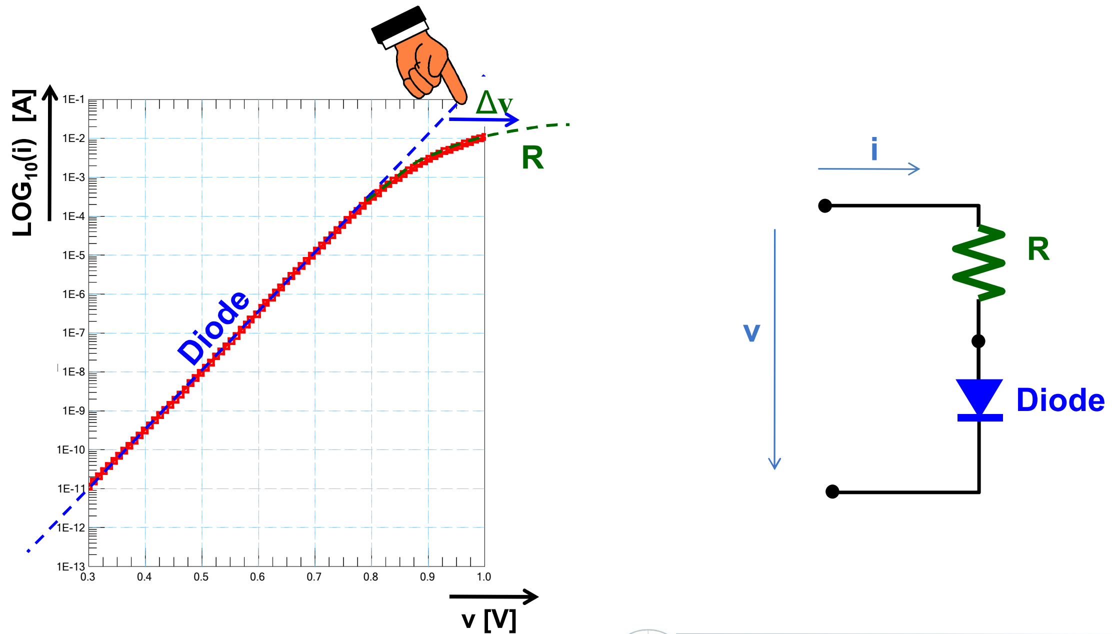
- taking the source code of the model and enhance it
- or perform a modeling using a sub-circuit with a combination of standard components

And like always, there are pro's and con's:
- an enhancement of the model equations -not an easy task, but elegant- by applying Verilog code may cause troubles related to using it in different simulators.
- sub-circuit modeling, will run in most simulators, but might sometimes be a compromise concerning model accuracy. Nevertheless, it is an important approach. This chapter features methods about how to compose such a sub-circuit.

Three Different Examples of Diode Modeling

A First Example: Standard Diode

at a given *current*, a *higher voltage drop* than what is predicted by the modeling: ⟶ the additional model component must be *in series*



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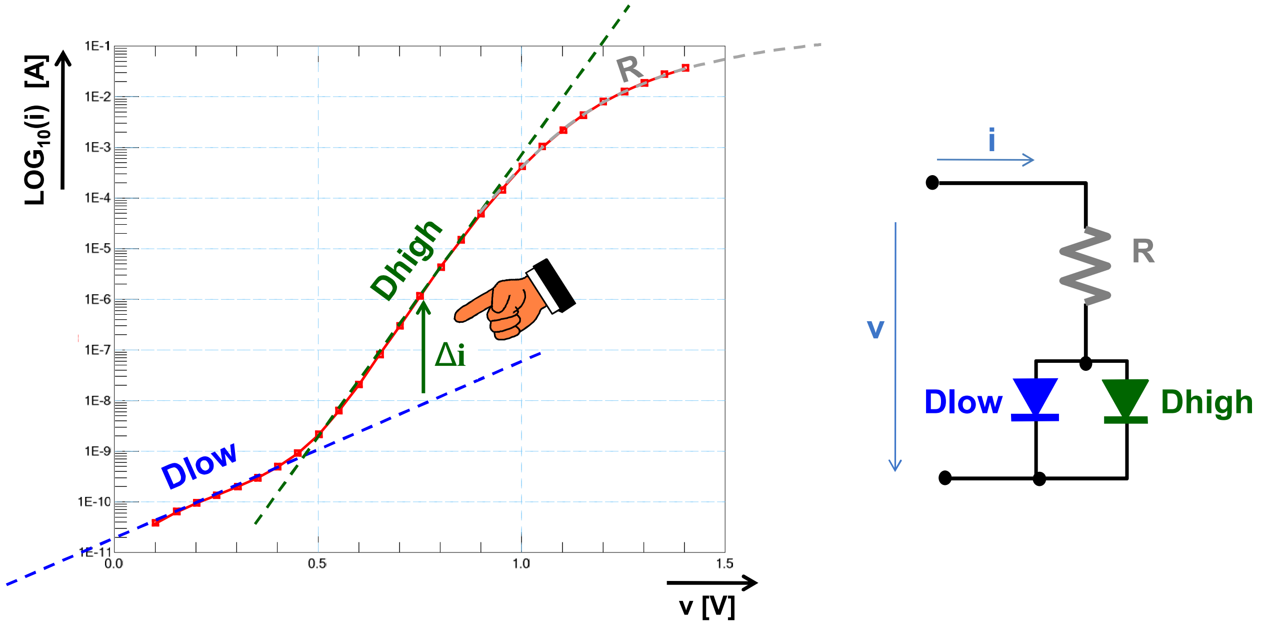
The first example of subcircuit modeling refers to a standard diode in DC forward operation.
The measurement result is given in a semi-logarithmic plot, with the y-axis representing the measured current 'LOG$_{10}$(i)' and the x-axis the stimulus voltage 'v'.
Following best-practice experience, the modeling starts at lowest bias voltage, and continues step-by-step up to the highest bias.
As shown above, the standard model 'Diode' with its equation i=IS*exp[v/(N*vt)-1] fits the measurement very well, up to v~0.8V. For higher bias voltage, the measured curve declines, while the so far developed model continues with its straight line. In other words, above v~0.8V, and at a given current 'i', there is a bigger voltage drop across the device than predicted by the standard diode model (dashed blue line).

A bigger voltage drop at a certain current means a second model connected in series with the existing one.

And, consequently, this 2nd model inside our subcircuit is a resistor (dashed green line).
The resulting model for this silicon diode is depicted above, on the right.

A Second Example: Diode with Recombination Range in DC Forward

at a given *voltage is* a higher *current* flowing than predicted
by the modeling: → the additional model component must be in *parallel*



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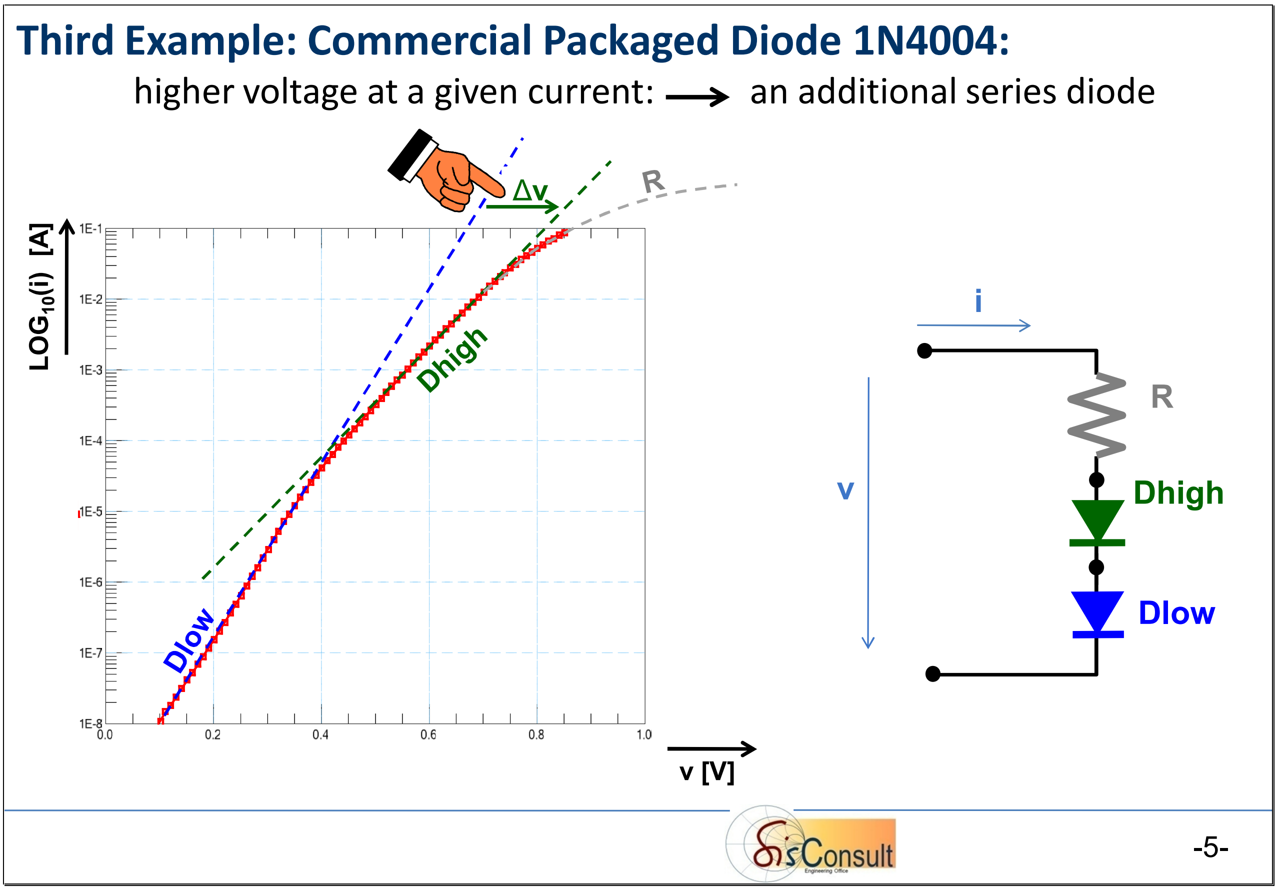
The second example is a diode exhibiting a lower slope for low bias voltage.
The modeling starts again at lowest bias voltage with the standard diode model, called 'Dlow'.
For higher bias voltage, above the knee, the measured curve becomes *steeper*, while the 'Dlow' model continues with its straight line: there is a bigger current (dashed green) flowing through the device than predicted by the standard diode model (dashed blue line).

<u>A bigger current flowing at a certain voltage means that the second model needs to be added in parallel with the existing one.</u>

Since this measurement region (~0.5V ... ~0.9V) is again represented by a straight line, this 2nd model is another ideal diode 'Dhigh' (dashed green line).
Finally, at high bias voltage, the diode resistor 'R' becomes dominant, like in the diode example before.
The resulting model for this GaAs diode is shown above, on the right.



The last example in this diode series is a commercial, packaged diode 1N4004.
This time, the DC forward measurement shows a *decline* in slope, well before the effect of the series resistor 'R'.
As always, the modeling starts at lowest bias voltage 'v' with the standard diode model, called Dlow.
For higher bias voltage, above the knee, the slope of the measured curve becomes *lower*: there is a bigger voltage drop (dashed green) across the device than predicted by the standard diode model (dashed blue line).

Once again: A bigger voltage drop at a certain current means to add a model segment in series with the existing one.

Since this measurement region (~0.5V ... ~0.7) is represented by a straight line, this 2nd model is once again an ideal diode 'Dhigh' (dashed green line).
Finally, like in the examples before, the diode resistor 'R' becomes dominant at high bias voltage.

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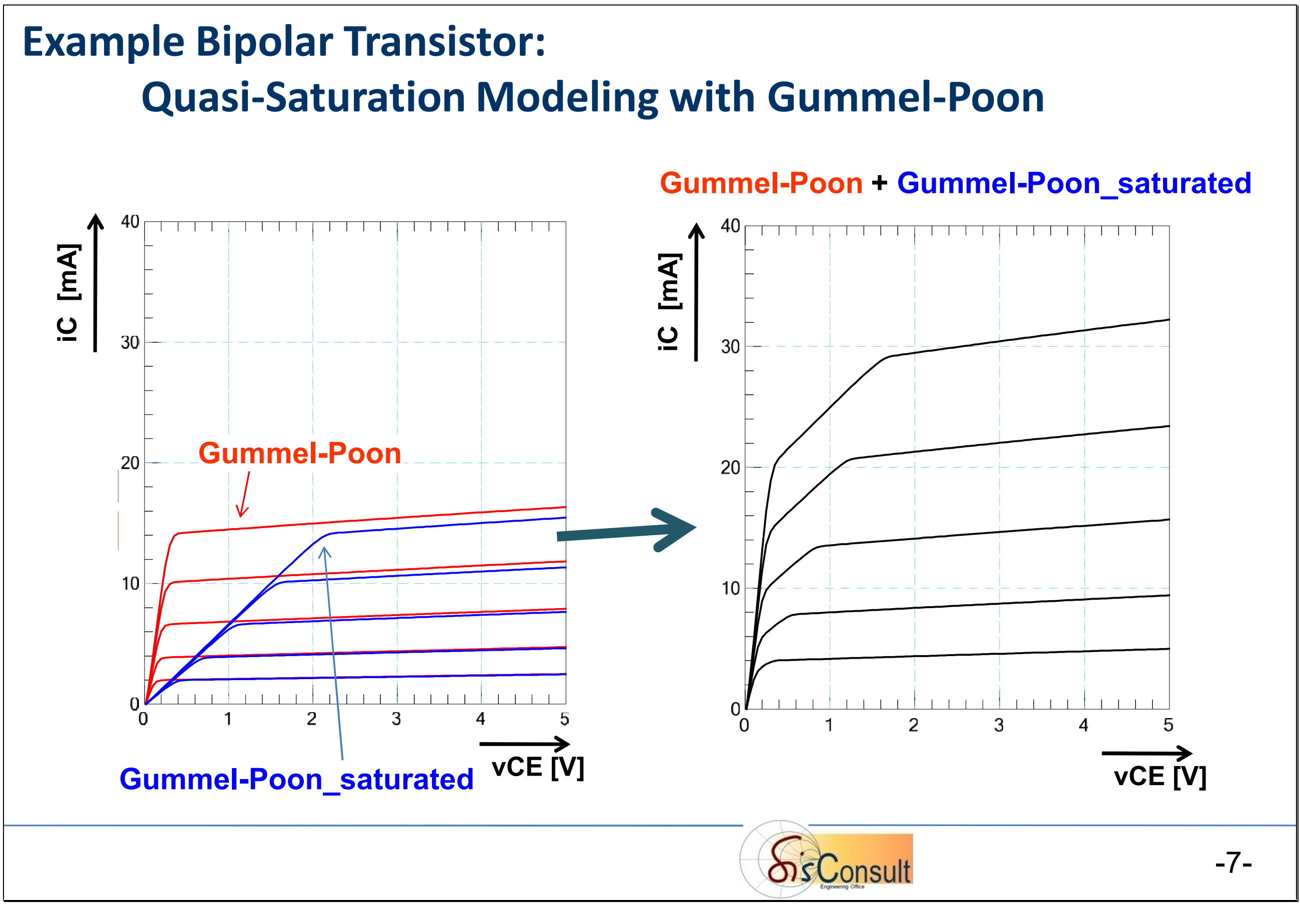
Example: Bipolar Transistor

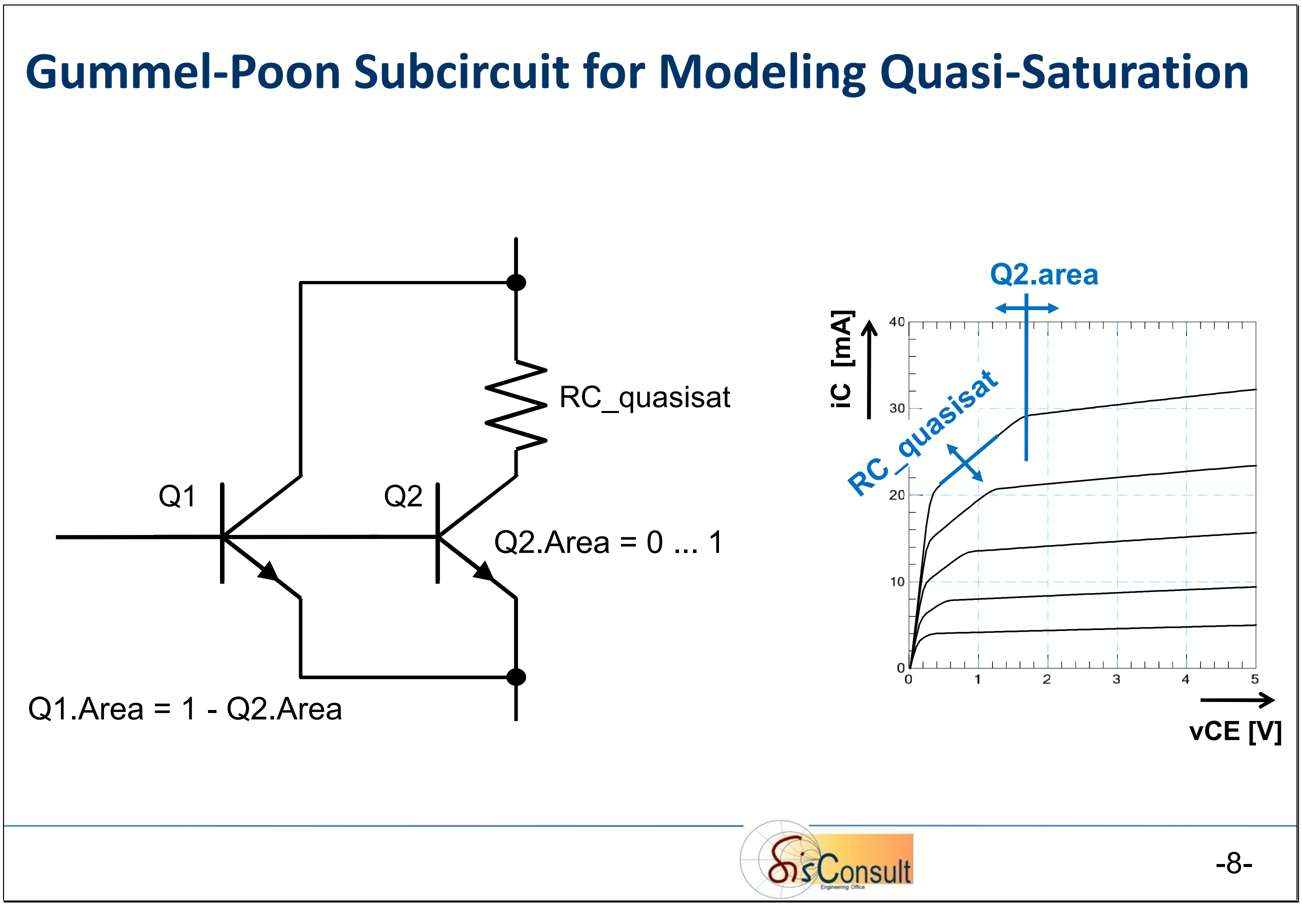


The sub-circuit modeling technique can be applied to many interesting modeling problems.

The Gummel-Poon model, for instance, is known for its limited fitting capabilities in the quasi-saturation range. However, when splitting the output characteristics of a bipolar transistor into two parallel and linked models (overlying both Collector currents), this problem can be solved:

- a first one (red) responsible for the output slope (Early effect),
- and another one (blue) for modeling the quasi-saturation.

See the next slide for the details.



In details:
the subcircuit consists of a single Gummel-Poon model, but applied to ***two*** transistor locations (instances 'Q1' and 'Q2'), plus a resistor 'RC_quasisat'. I.e. the same set of Gummel-Poon parameters is shared by the two transistors, but their area parameters 'Area' are different and can be used for distributing the Collector current among them, while resistor 'RC_quasisat' can drive 'Q2' in deep saturation.

For the modeling of iC(vCE), the instance parameter 'Q2.area' determines the transition from the quasi-saturation to the non-saturated range, while the resistor 'RC_quasisat' affects the slope in quasi-saturation, as depicted above.

The subcircuit in 'Spice' netlist syntax (ASCII) is:

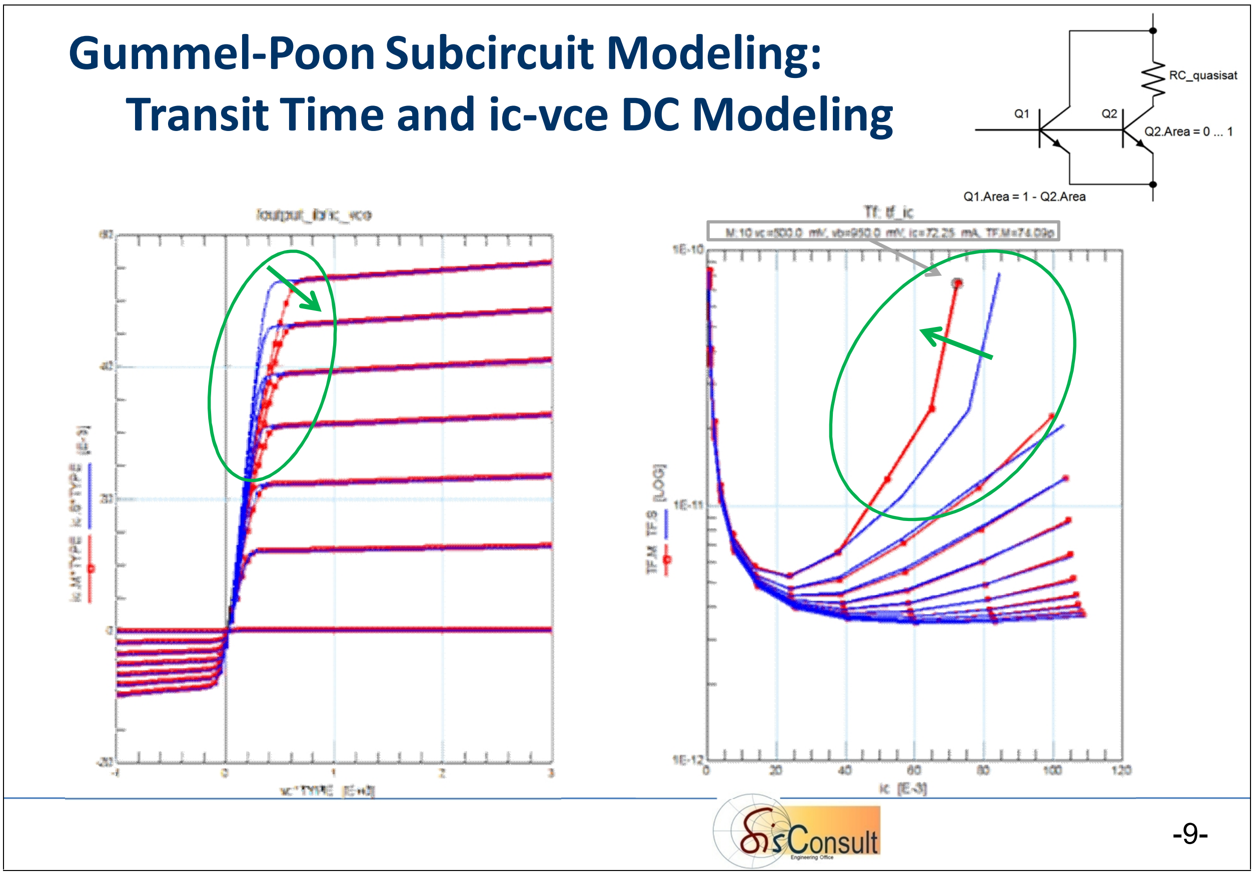
```
.SUBCKT GP_and_Quasisat  C=1 B=2 E=3

Q1            1  2  3  MAIN  Area = 0.6524
Q2           11  2  3  MAIN  Area = 0.3476
RC_quasisat     1 11  100

* Common Gummel-Poon ModelCard
.MODEL MAIN NPN  IS=3.596E-15  NF=1.017 ISE=3.7E-15 NE = 1.509
+ BF=281.2 IKF=0.3173 VAF=25.35    ...    ...

.ENDS
```

In order to verify the DC subcircuit modeling also in the S-parameter domain, the slide below investigates the effect of changing the Area and RC_quasisat parameters in the transit time modeling plot. As can be seen on the plot on the right, it only affects the trace for highest DC biasing, what is a correct behavior. It does not perturb the fitting in the other biasing ranges.



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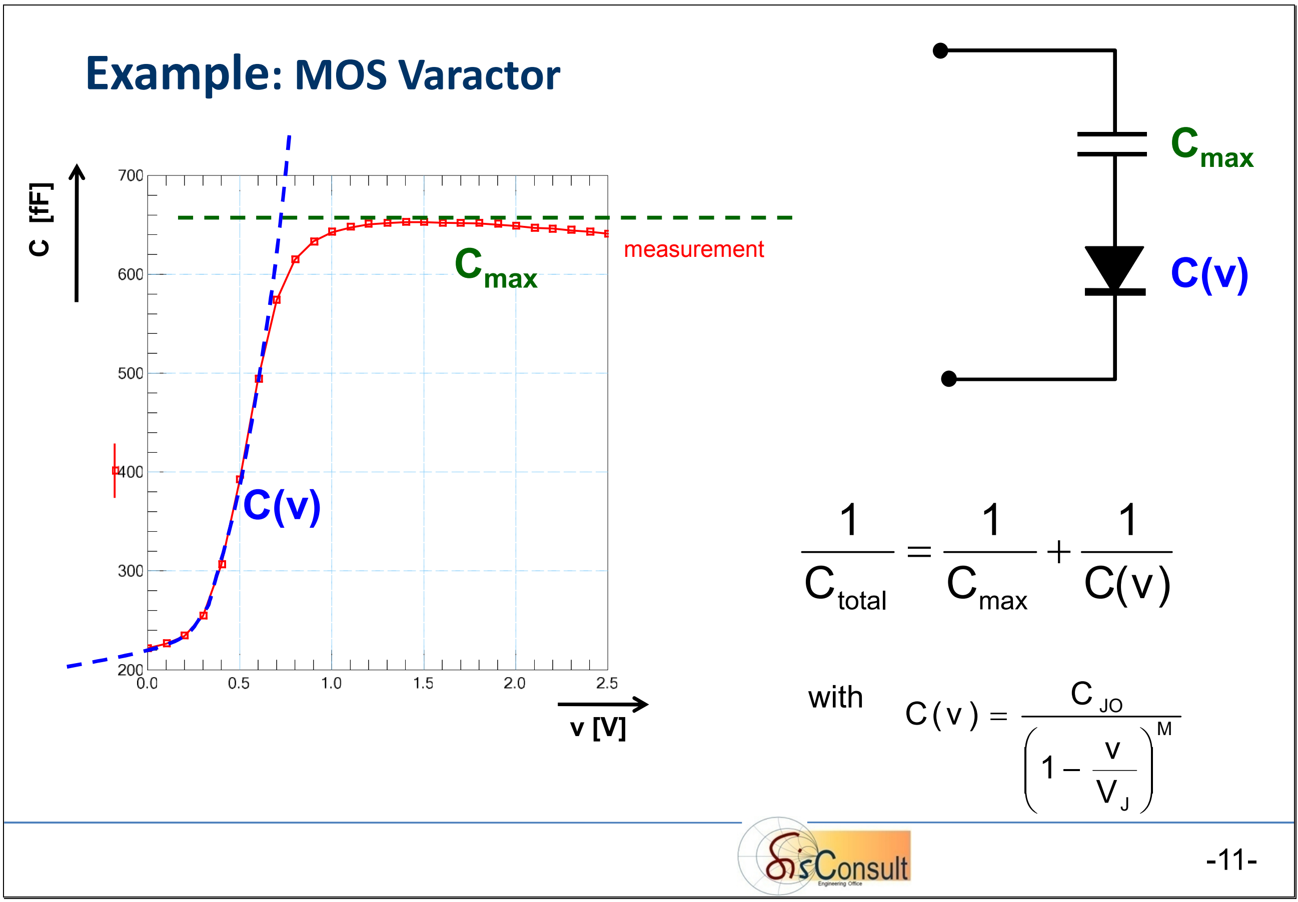
Example: MOS Varactor

This example shows the sub-circuit modeling of a MOS varactor.



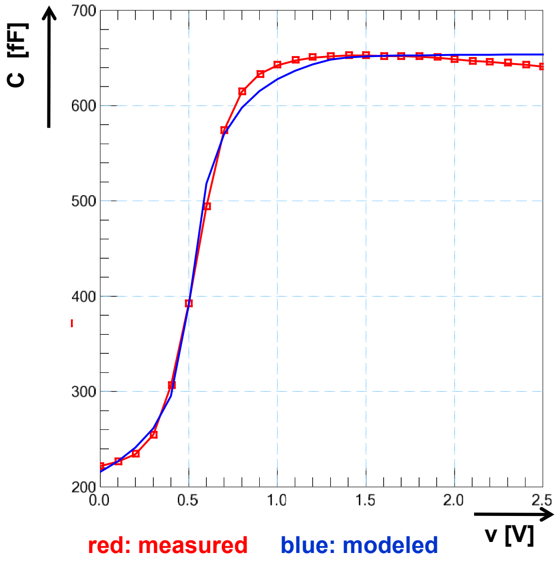
The measurement (red) can be split into two segments: a diode CV-characteristic (blue) and a fixed, voltage-independent range (green).

The modeling is again performed 'from the left to the right', i.e. with increasing bias voltage.
The diode CV parameter 'C_{JO}' is equal to the y-intercept C(v=0), 'M' fits the slope and 'V_J' the range v>0.

For v>0.7V, this model would 'explode', resulting in a short circuit.
This is prevented by adding a capacitor 'C_{max}' in series, what corresponds physically to the gate oxide of the MOS varactor.

Note: it is recommended to add a big resistor (e.g. 1GOhm) to ground between capacitance Cmax and the Diode model, for improved Spice convergence.

Fitted MOS Varactor




```
.SUBCKT GateOxide 1 2

Cmax     1 12 6.92E-13
Diode   12 2  MAIN
Rdummy   1 12 1E+6

*model card
.MODEL MAIN D IS=1E-30  N=1
+  CJO=3.136E-13  M=0.345
+  VJ=0.524  FC=0.95

.ENDS
```

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Note: Rdummy in parallel to Cmax has been added to provide a minimum DC current to the diode model for improved Spice simulation convergence.

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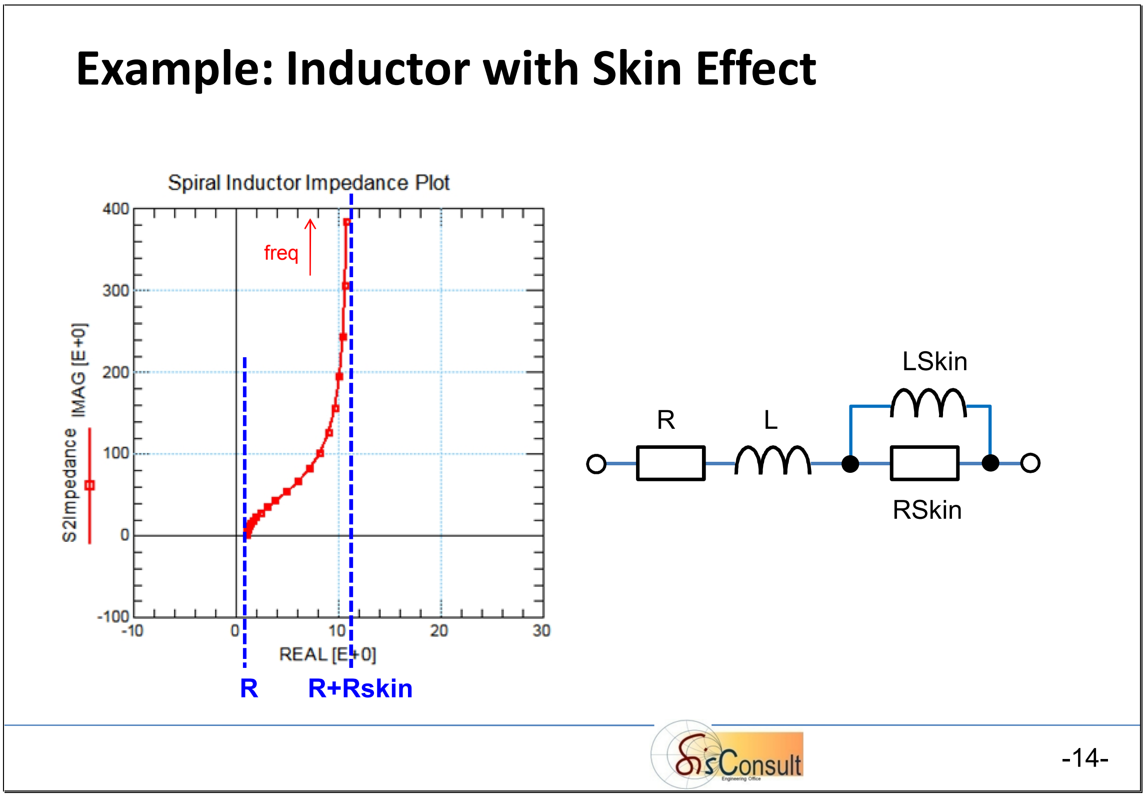
Example: Inductor with Skin Effect

And here the last of the series of sub-circuit Spice modeling examples:



At lowest frequencies, the model consisting of inductor L and its series resistor R fits the measurement.

With increasing frequency, the skin effect shows up as an increase of the series resistance.
This can be modeled using a series resistor RSkin, in parallel with an inductor LSkin, which shorts RSkin at DC, and leaves it open for infinite frequency.

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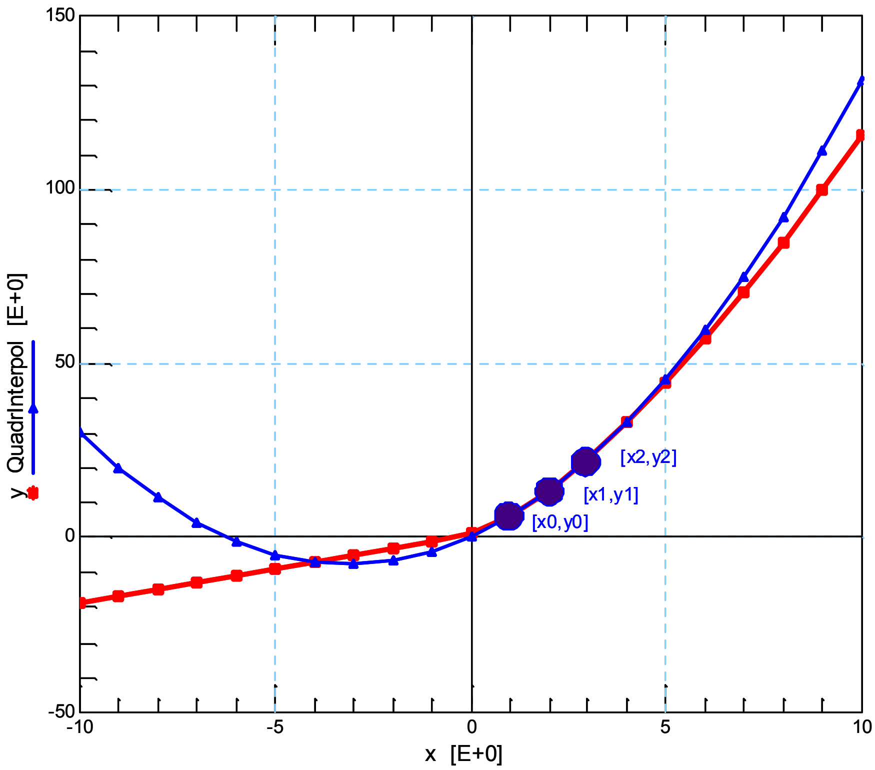
Data Interpolation

This chapter features the fitting of a quadratic function to three data points:

$$y = a + b*x + c*x^2$$



For three data points [x0, y0], [x1, y1], [x2, y2], we start with a set of 3 quadratic equations

$$y0 = a + b * x0 + c * x0^2 \quad (1)$$

$$y1 = a + b * x1 + c * x1^2 \quad (2)$$

$$y2 = a + b * x2 + c * x2^2 \quad (3)$$

Goal: We need to get the coefficients a, b and c as a function of xi and yi

From (1):

$$a = y0 - b * x0 - c * x0^2 \quad (4)$$

(4) into (2):

$$y1 = y0 - b * x0 - c * x0^2 + b * x1 + c * x1^2 \quad (5)$$

In the same way, from (3):

$$\boxed{a = y2 - b * x2 - c * x2^2} \quad (6)$$ **what is the first fitting parameter**

(6) into (2):

$$y1 = y2 - b * x2 - c * x2^2 + b * x1 + c * x1^2$$

$$b * (x1 - x2) = y1 - y2 + c * \left(x2^2 - x1^2\right)$$

$$\boxed{b = \frac{y1 - y2 + c * \left(x2^2 - x1^2\right)}{(x1 - x2)}}$$ **what is the second fitting parameter** (7)

Finally, (7) into (5):

$$y1 = y0 + (x1 - x0) * \frac{y1 - y2 + c * \left(x2^2 - x1^2\right)}{(x1 - x2)} + c * \left(x1^2 - x0^2\right)$$

$$(y1 - y0) * (x1 - x2) - (x1 - x0) * (y1 - y2) = c * \left(x1^2 - x0^2\right) * (x1 - x2) - c * (x1 - x0) * \left(x1^2 - x2^2\right)$$

$$\boxed{c = \frac{(y1 - y0) * (x1 - x2) - (x1 - x0) * (y1 - y2)}{\left(x1^2 - x0^2\right) * (x1 - x2) - (x1 - x0) * \left(x1^2 - x2^2\right)}}$$ **what is the third fitting parameter** (8)

In other words:

Equations (6), (7) and (8) represent the equations for the polynomial coefficients a, b and c.

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