TwoPort Matrices and their Impact on Device Measurements and Modeling

Abstract:
With the introduction of the S-parameters in the 1960s, the two-port theory of the 1920ies gained an important application also in the high-frequency range, since it is able to separate the netlist components from the overall measurements. This is a property that S-parameters alone do not provide.

Z, Y, H, A matrices, calculated from S-parameters, contribute to the accuracy of measurements and device modeling in terms of de-embedding, device modeling and verification of the achieved fit.

This paper presents a summary of relevant relationships, applications and best practices for these matrices.

Keywords:
TwoPort Matrix Definitions, Important Matrix Elements for Device Modeling,

Contents:

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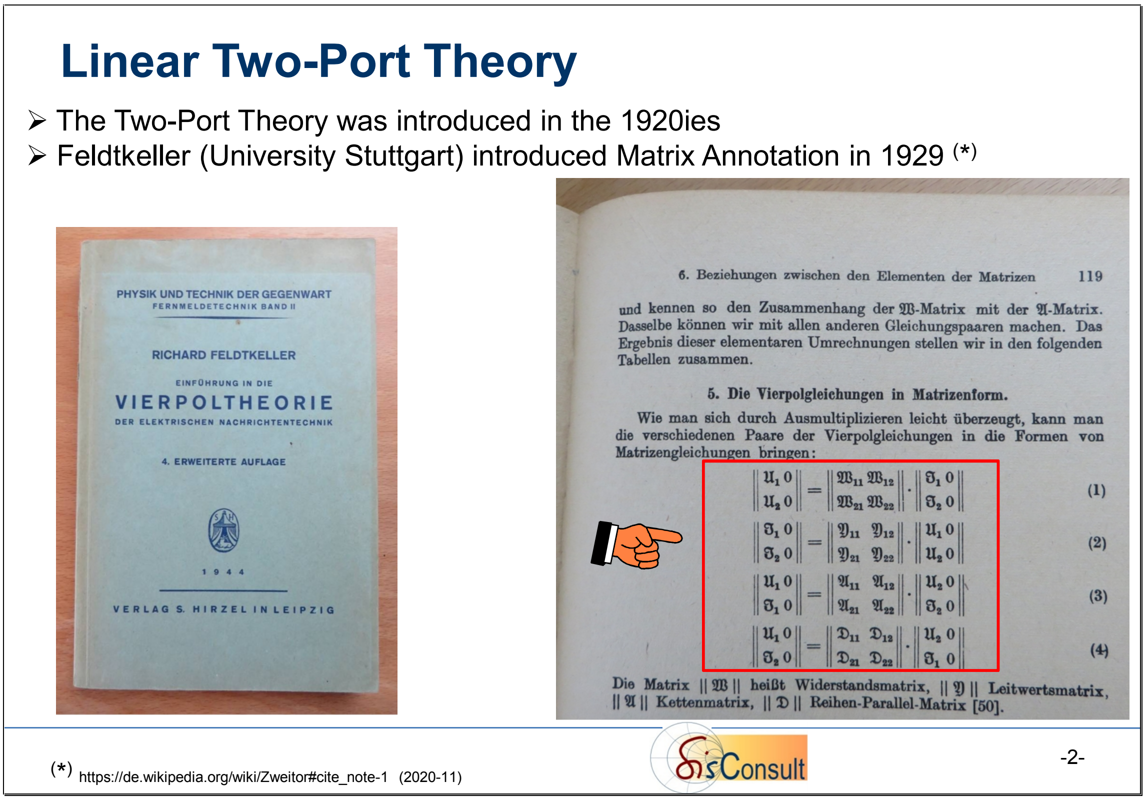
A Bit of History



The classical, linear Two-Port Theory relates input and output currents and voltages by linear matrix operations.
For example, for the Z matrix, the Two-Port is stimulated by a current source into each port, and the voltages at the ports are measured.
For the Y matrix, the stimulation is by a voltage source at each of the ports, and the currents into the ports are measured.

NOTICE:
CAUTION: the Y-matrix is not defined for an impedance-to-ground-only Two-Port like this:

```
o---o---o
    ⊥
  |   | Z
    T
o---o---o
```

and the Z-matrix is not defined for a series-impedance-only Two-Port like this:

```
         ___
o---|___|---o
      Z
o-----------o
```

High-Frequency Measurement Problems

Drawbacks of ZYHA Matrix Theory for High Frequency Measurements:

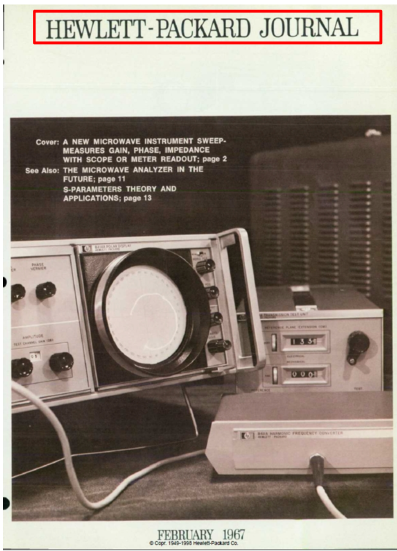
- to measure the matrix parameters,
  the Two-Port's input and output needs to be connected to ideal OPENs and SHORTs.
- at RF frequencies, OPENs and SHORTs are not ideal,
  but correspond rather to inductances and capacitances.
- additionally, even when such ideal terminations would be available and applied,
  the device may behave non-linear and oscillate.

S-Parameters, introduced in the 1960ies, are measured

- **by imbedding the device, on both sides,**
  into a well-defined 'Characteristic Impedance Z0',
- **and by calculating the relationship of reflected or transmitted 'Power Waves'**
  to the incident 'Power Wave' ($\sqrt{\text{Watt}}$).
- This realistic impedance Z0 also reduces the chance for device oscillations drastically.

... and since we have been talking about history ...

Introduction of S-Parameters in the 1960ies



HEWLETT PACKARD Application Note 95-1

S-Parameter Techniques for Faster, More Accurate Network Design

February 1967

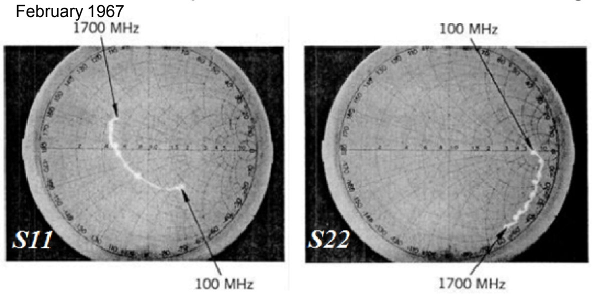


Fig. 4. S parameters of 2N3478 transistor in common-emitter configuration, measured by -hp- Model 8410A Network Analyzer.

In the industry, measuring S-Parameters began in the 1960ies, mainly supported by Hewlett-Packard and the introduction of its Network Analyzers. Corresponding publications appeared in the HP Technical Journal, and also by Application Notes. Above, on the right, is a screenshot of the 1967 HP Application Note 95-1.

In 1997, this highly recommendable Application Note has been completely updated by HP, covering S-Parameters from introduction up to expert level.

It is available at

http://www.agilent.com/find/eesof-an95-1   (as of Dec.2020).

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Overview Matrix Conversions

CONVERTING THE EASY-TO-MEASURE S-PARAMETERS TO CLASSICAL MATRICES

$\underline{S}$ to $\underline{Z}$ Matrix Conversion:

$$\underline{Z} = Z_0 \cdot \frac{E + \underline{S}}{E - \underline{S}}$$

$\underline{Z}$ to $\underline{S}$ Matrix Conversion:

$$\underline{S} = \frac{\underline{Z} - Z_0 \cdot E}{\underline{Z} + Z_0 \cdot E}$$

$\underline{S}$ to $\underline{Y}$ Matrix Conversion:

$$\underline{Y} = \frac{E - \underline{S}}{Z_0 \cdot (E + \underline{S})}$$

$\underline{Y}$ to $\underline{S}$ Matrix Conversion:

$$\underline{S} = \frac{E - Z_0 \cdot \underline{Y}}{E + Z_0 \cdot \underline{Y}}$$

$$\underline{S} = \begin{pmatrix} S_{11} & .. & S_{1N} \\ .. & \ddots & .. \\ S_{M1} & .. & S_{MN} \end{pmatrix}$$

$$\underline{Z} = \begin{pmatrix} Z_{11} & .. & Z_{1N} \\ .. & \ddots & .. \\ Z_{M1} & .. & Z_{MN} \end{pmatrix}$$

$$\underline{Y} = \begin{pmatrix} Y_{11} & .. & Y_{1N} \\ .. & \ddots & .. \\ Y_{M1} & .. & Y_{MN} \end{pmatrix}$$

$\underline{Z}$ to $\underline{Y}$ Matrix Conversion:

$\underline{Y} = 1/\underline{Z}$

$\underline{Y}$ to $\underline{Z}$ Matrix Conversion:

$\underline{Z} = 1/\underline{Y}$

Pre-Requisite:

- Z_0, the characteristic impedance of the S-parameter measurement, is identical on all ports, and Z_0 = REAL

Notice:

- E is the identity matrix. It can be calculated the easiest way from any N-Port matrix by setting it to exponential 0: **E = (Any_N_Port_Matrix)^0**

These formulas convert matrices with any number of ports.

Special acknowledgements to Dave van Goor and Luuk Tiemeijer, NXP Eindhoven (March 2007)

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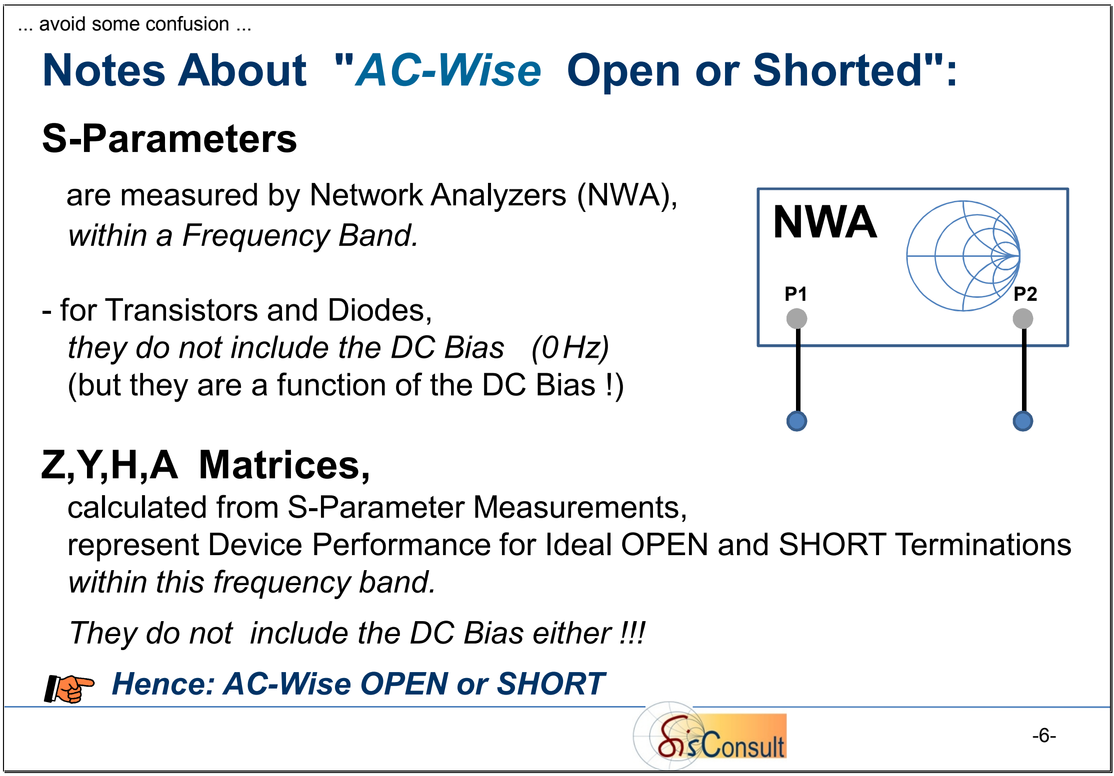
PEL Programs for IC-CAP Users:

```
Path2Data="/myModel/myDut/mySetup/mySpar_Output"


!S to Z
!=========================================
 Z0 = 50
 tmpS = DATASET("quiet "&VAL$(Path2Data))
 E = tmpS^0 !define an identity matrix
 tmpZ = Z0*(E+tmpS)//(E-tmpS)
 RETURN tmpZ


!Z to S
!=========================================
 Z0 = 50
 tmpZ = DATASET("quiet "&VAL$(Path2ZData))
 E = tmpZ^0 !define an identity matrix
  tmpS = (tmpZ-Z0*E)//(tmpZ+Z0*E)
 RETURN tmpS


!S to Y
!=========================================
 Z0 = 50
 tmpS = DATASET("quiet "&VAL$(Path2SData))
 E = tmpS^0 !define an identity matrix
 tmpY = 1//Z0*(E-tmpS)//(E+tmpS)
 RETURN tmpY
```

```
!Y to S
!========================================
 Z0 = 50
 tmpY = DATASET("quiet "&VAL$(Path2YData))
 E = tmpY^0 !define an identity matrix
 tmpS = (E-Z0*tmpY)//(E+Z0*tmpY)
 RETURN tmpS
```

```
!Z to Y
!========================================
 tmpZ = DATASET("quiet "&VAL$(Path2ZData))
  tmpY = tmpZ^-1
 RETURN tmpY
```

```
!Y to Z
!========================================
 tmpY = DATASET("quiet "&VAL$(Path2YData))
  tmpZ = tmpY^-1
 RETURN tmpZ
```



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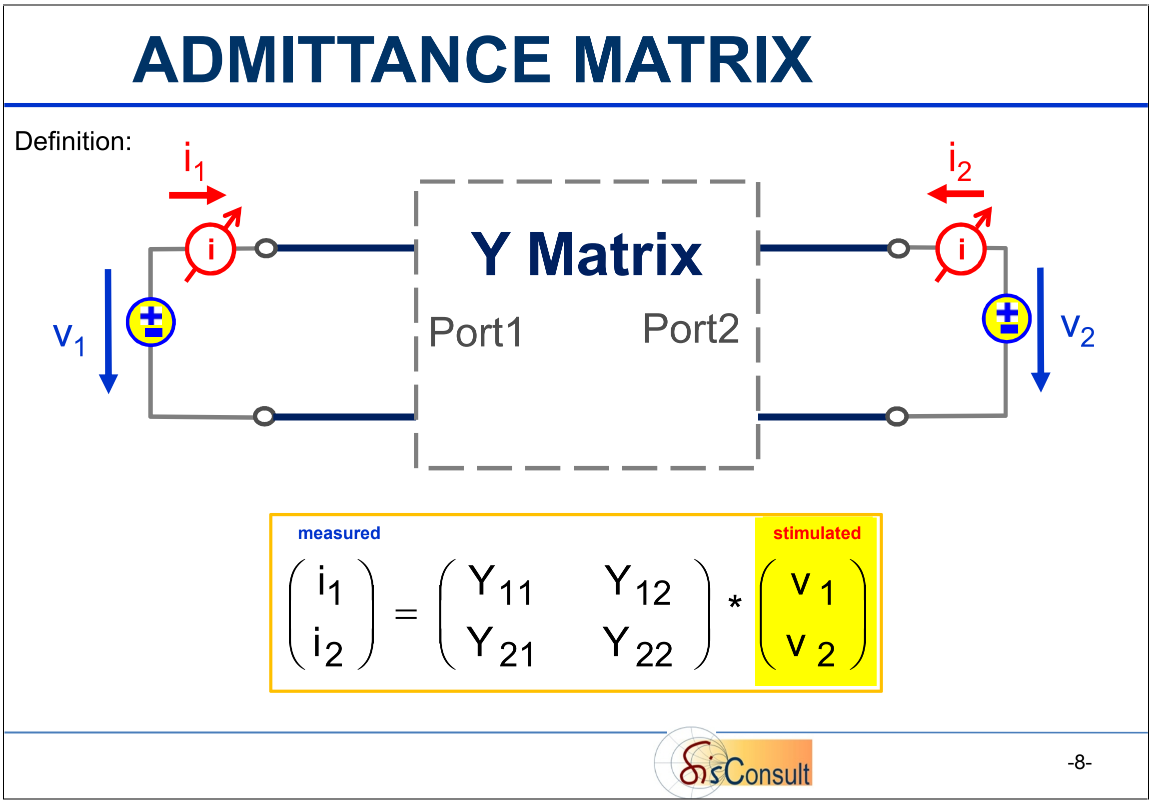
A Walk through the Properties of the TwoPort Matrices and their Applications to Device Modeling



The Admittance or Y-Matrix is stimulated by a voltage at each port, and the currents are measured.

Important Y-Matrix Elements:

measured | stimulated

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} * \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$\frac{1}{Y11}$ = v1/i1 @v2=0
= **Total** Input Impedance with **Port2 SHORTED**

$\frac{1}{Y22}$ = v2/i2 @v1=0
= **Total** Output Impedance with **Port1 SHORTED**

Application: Passive Components Modeling

Important Y-Matrix Elements (cont'd):

measured | stimulated

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} * \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

Applications:

Passive Components Modeling → $-\frac{1}{Y21}$ = v1/(-i2) @v2=0
Impedance from Port1 **to** Port2 with **Port2 SHORTED**

gm of FETs and HEMTs → **Y21** = i2/v1 = gm @v2=0
Trans-Conductance from Port1 **to** Port2 with **Port2 SHORTED**

Alternatively, to avoid the condition of AC-wise shorted Port2, apply PI schematic modeling (see further below)

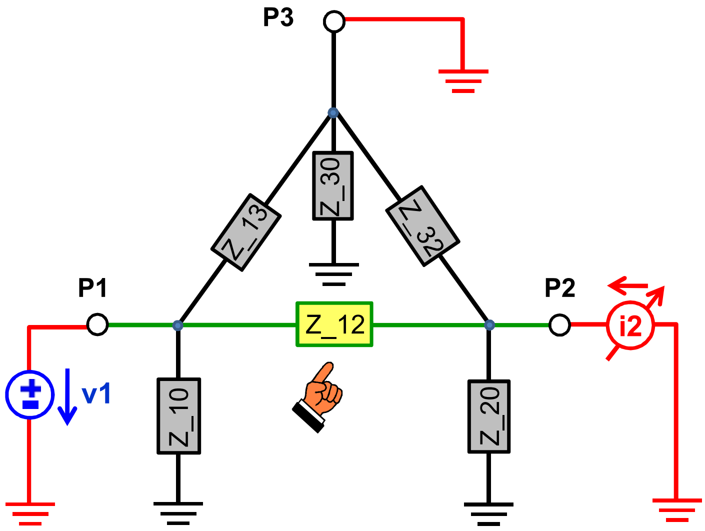
Two More Y-Matrix Application Examples:

- ➢ **Impedance Measurements of Multi-Ports**
- ➢ **Transistor Inner PI Schematic Modeling**

Y-Matrix Application Example:

How to Calculate the Individual Branch Impedances of a 3 Port

Port1-to-Port2-Impedance Z_12



From an inspection of the 3-Port Y-Matrix definition:

$$\begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{pmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

apply a SHORT to Port2 and Port3, stimulate a voltage at Port1, measure the current at Port2 and calculate:

$$Z\_12 = \frac{v1}{-i2} = -(Y21)^{-1}$$

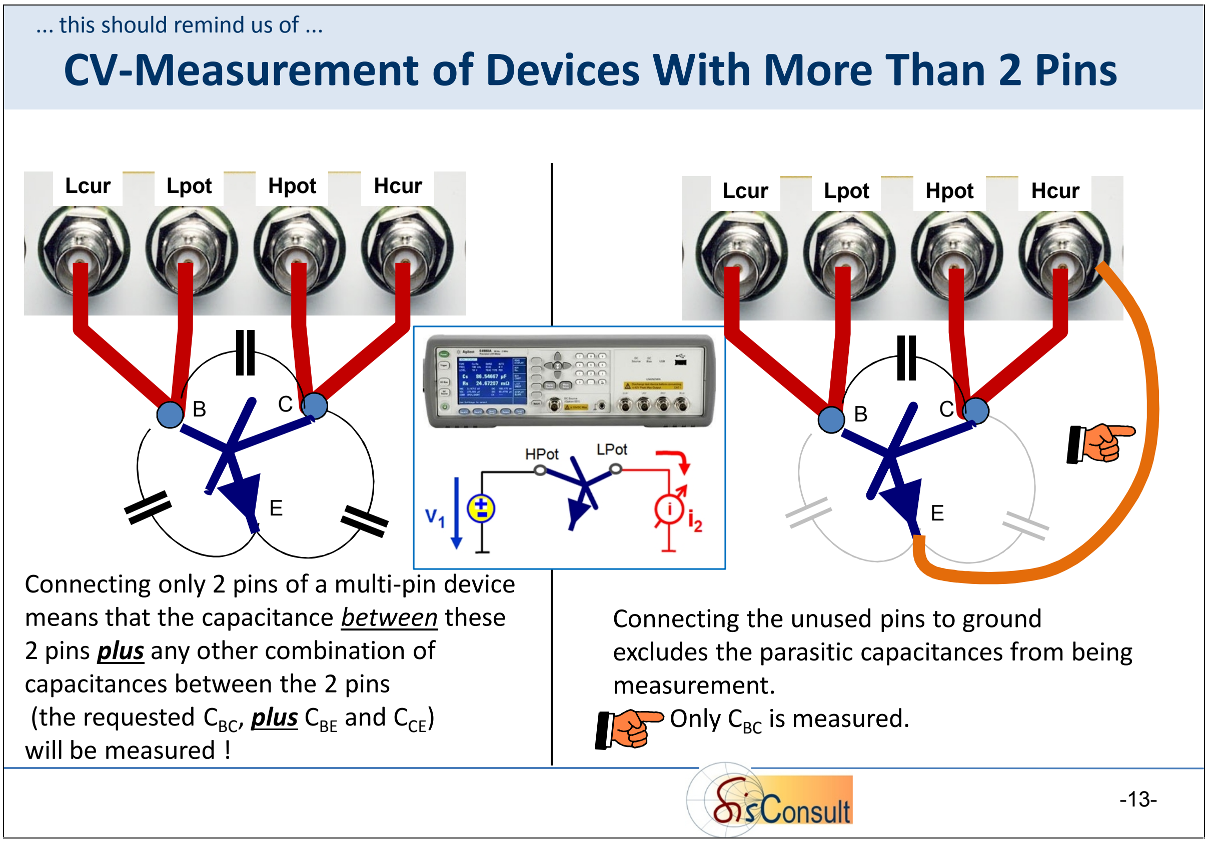
Note:
the Y-Matrix indexing is $Admittance_{to\ from}$

e.g. the Admittance from Port1 to Port2 is Y21

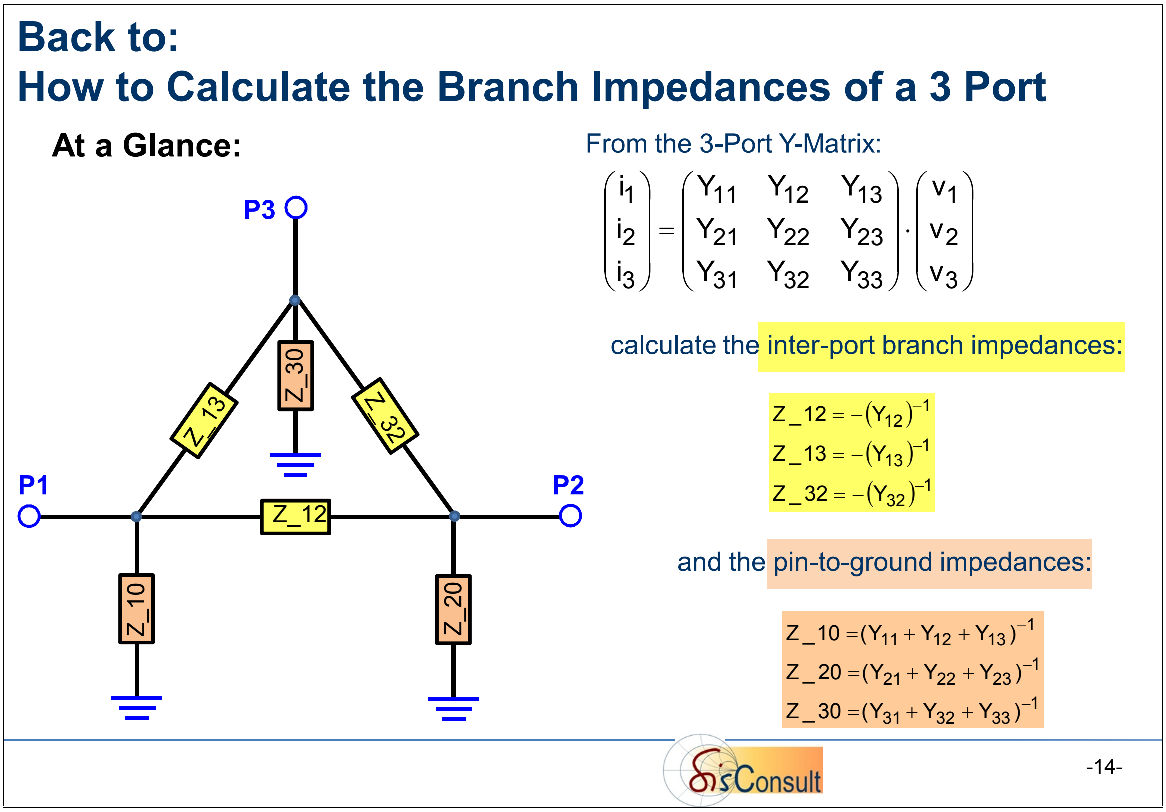
Notes:

- ➢ Compare the Y-Matrix indexing with the S-Parameter indexing:
  S21: transmission from Port1 to Port2

A quick intermezzo:

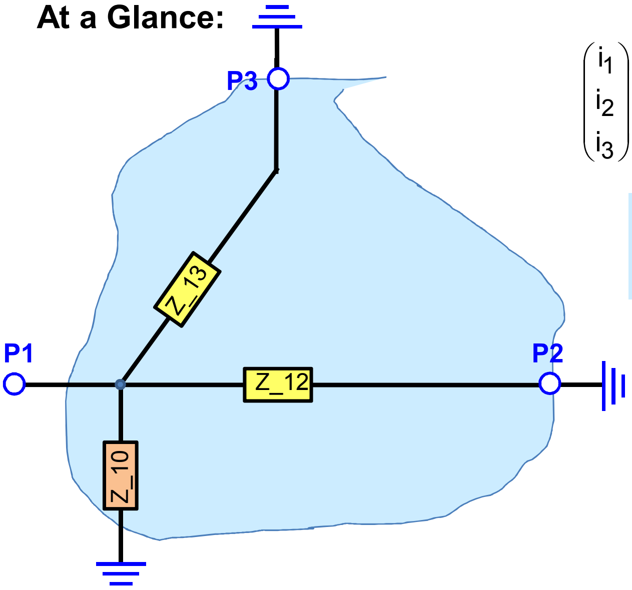


➢Notice: This is a very often overlooked detail when performing CV measurements: just the 2 pins are connected to the instrument, and the measurement result is interpreted as only the capacitance between these 2 pins, overlooking the parasitic add-on effect of the remaining capacitances.



How to Calculate the Branch Impedances of a 3 Port (cont'd)

At a Glance:



$$\begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{pmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

- The total impedance at P1, is 1/Y11 (P2 and P3 grounded)

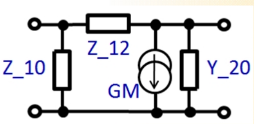
and so on:

- At P2, with P1 and P3 grounded, the total impedance is 1/Y22
- And at P3, with P1 and P2 grounded, it is 1/Y33

... and another Y-Matrix Application Example:

MOS/MESFET/HEMT: Transistor Inner PI Schematic Modeling

1. Convert de-embedded S-parameters to Z, and strip-off the remaining external inductors.
2. To obtain the Inner PI Schematic, convert further to Y-parameters, and calculate



Z_10 = 1 / (Y.11 + Y.12) — Impedance Port1 → GND

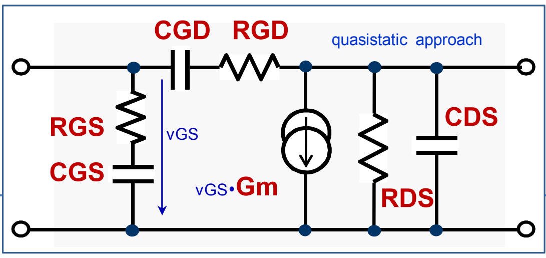
Z_12 = 1 / (-Y.12) — Impedance Port1 → Port2

GM = Y.21 - Y.12 = GM • $e^{-j \cdot 2PI \cdot freq \cdot TAU}$ — Voltage → Current Amplification

Y_20 = Y.22 + Y.12 — Admittance Port2 → GND

3. Finally, get

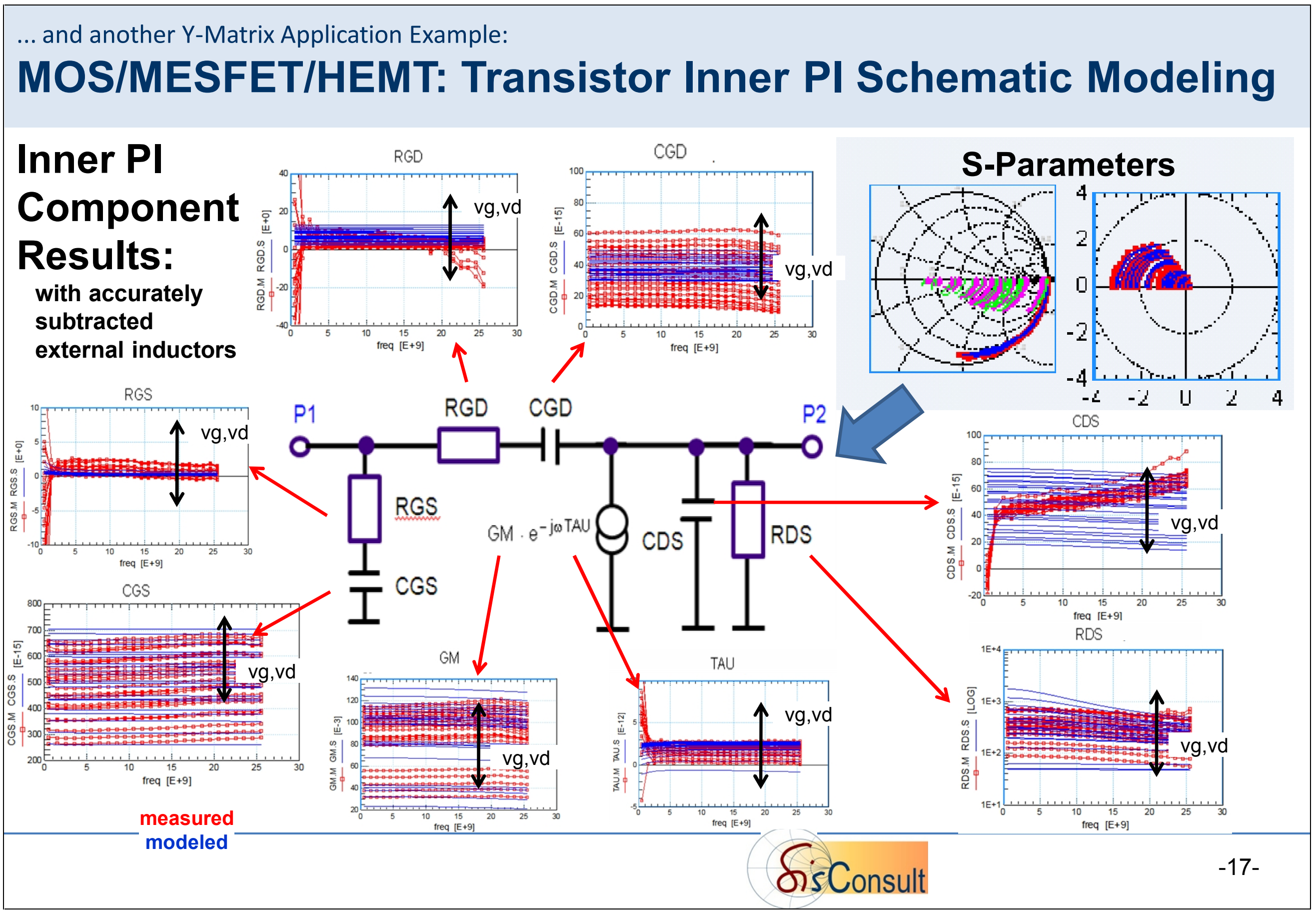
RGS = REAL(Z_10)	CGS = - 1 / (IMAG(Z_10) • 2PI • freq)
RGD = REAL(Z_12)	CGD = - 1 / (IMAG(Z_12) • 2PI • freq)
GM = MAG(GM)	TAU = - PHASE(GM) / (2PI • freq)
RDS = 1 / (REAL(Y_20))	CDS = IMAG(Y_20) / (2PI • freq)



Note: if the values of the off-stripped **external inductors** are correct, the inner-PI schematic components become frequency-independent. Only DC bias dependent.

This corresponds to the frequency independence of the inner-PI components of models like EEHEMT, Angelov, ASM-HEMT and also of the MOS models.

The Inner-PI-Modeling is the most detailed interpretation of Y-Parameters for transistors with negligible or stripped-off RG. Therefore, it works best for MOS, MESFETs and HEMTs.
In addition, as described above, it can also be applied to identify the values of the external inductors.



A typical measurement verification result, with correctly stripped-off external inductors: only very little dependency of the Inner-PI-schematic components versus frequency.

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IMPEDANCE MATRIX

Definition:

measured | stimulated

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} * \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}$$

Note: $Z = [Y]^{-1}$

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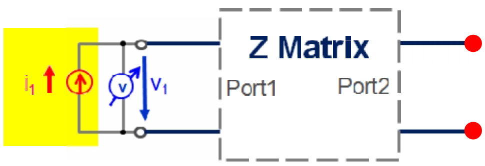
The Impedance or Z-Matrix is stimulated by a current at each port, and the voltages are measured.

Important Z-Matrix Elements:

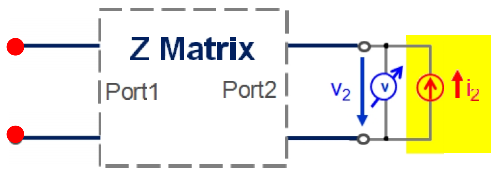
measured | stimulated

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} * \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}$$

Z11 = v1/i1 @i2=0
= Input Impedance with **Port2 OPEN**



Z22 = v2/i2 @i1=0
= Output Impedance with **Port1 OPEN**



Application: Verification of Passive Components Modeling

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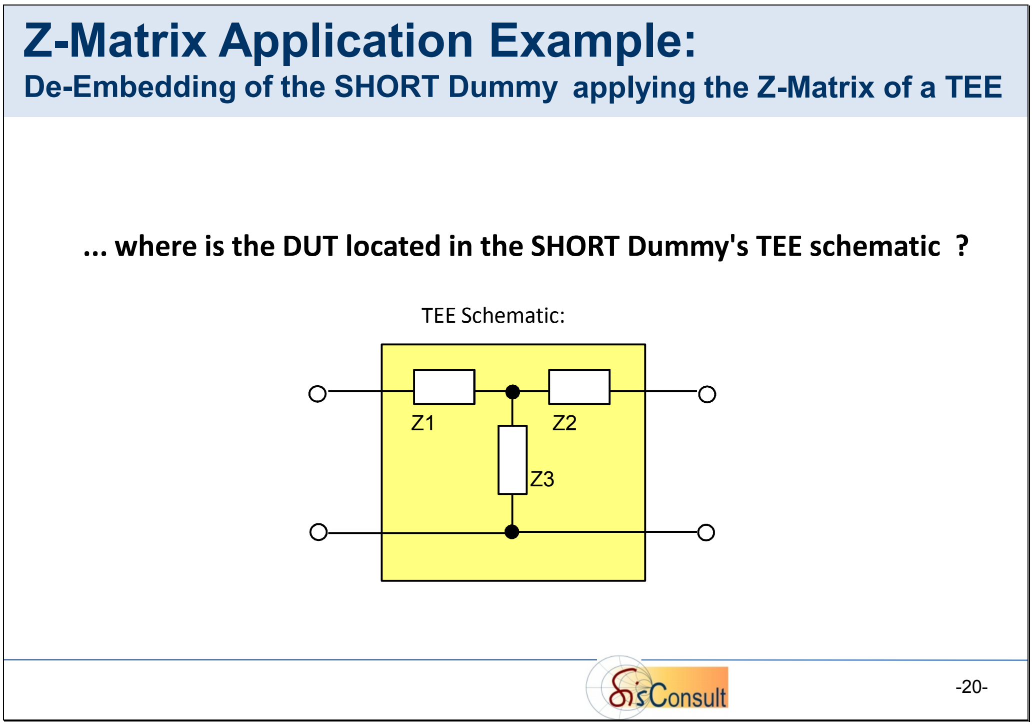
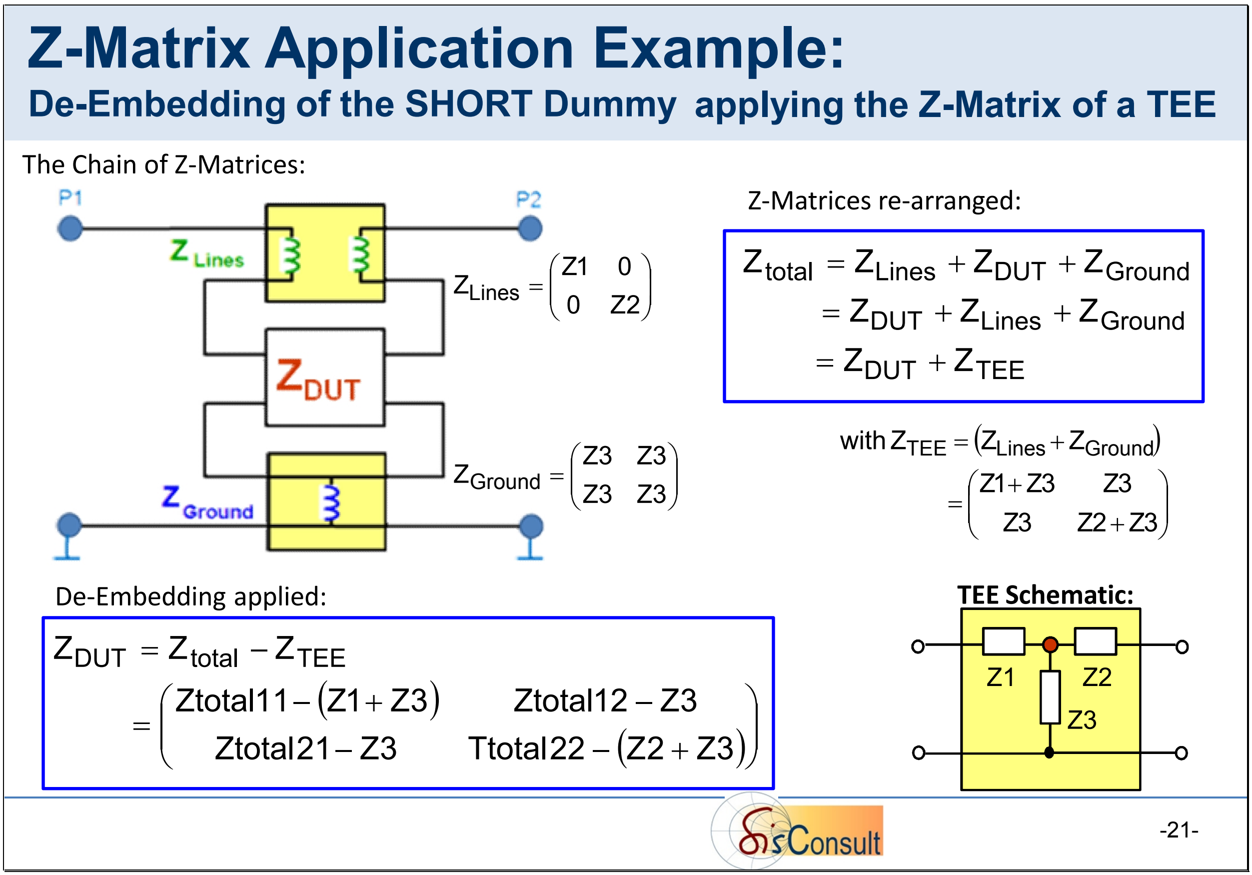


During S-Parameter de-embedding, everybody applies a TEE schematic for the SHORT Dummy off-stripping. Obviously, the DUT is replaced by the middle connection point of the three impedances. Let's see how to proof this.



The DUT is located at the inner node (!) of the SHORT Dummy's TEE Schematic, because the mathematical commutative law states that the addition of matrices is independent of the sequence. This combines Z_{Lines} and Z_{Ground} to Z_{TEE}.

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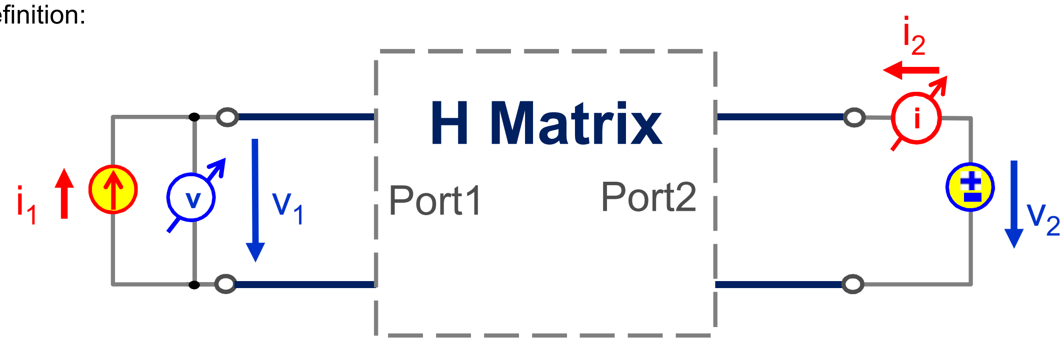
HYBRID MATRIX

Definition:



measured / stimulated

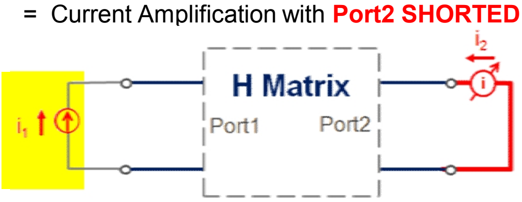
$$\begin{pmatrix} v_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} * \begin{pmatrix} i_1 \\ v_2 \end{pmatrix}$$

Important H-Matrix Elements:

measured / stimulated

$$\begin{pmatrix} v_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} \\ \mathbf{H_{21}} & H_{22} \end{pmatrix} * \begin{pmatrix} i_1 \\ v_2 \end{pmatrix}$$

H21 = i2/i1 @v2=0
= Current Amplification with **Port2 SHORTED**



Application:

Beta$_{AC}$ Modeling of HBT and Bipolar Transistors

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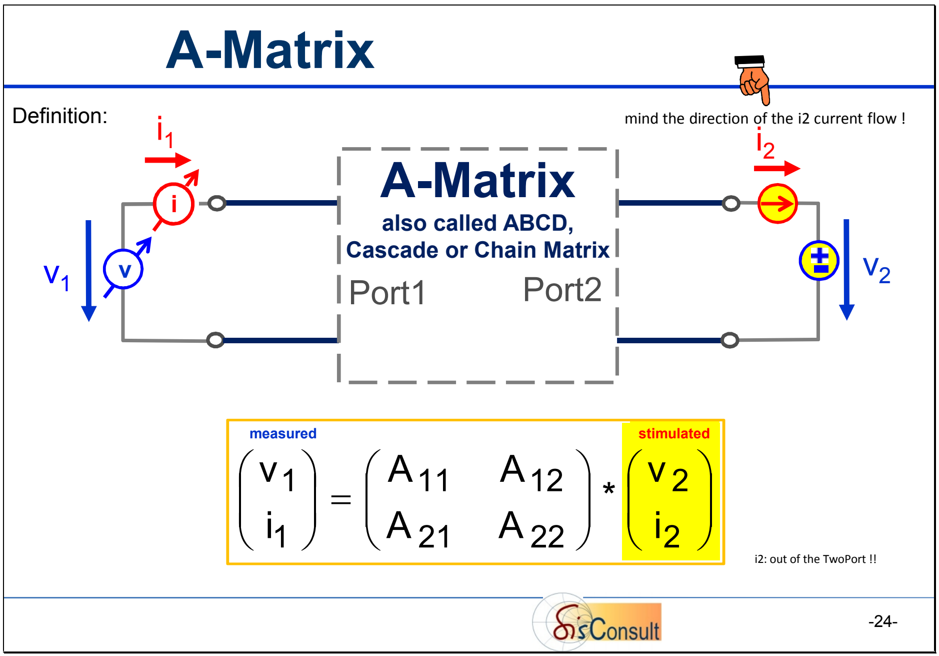


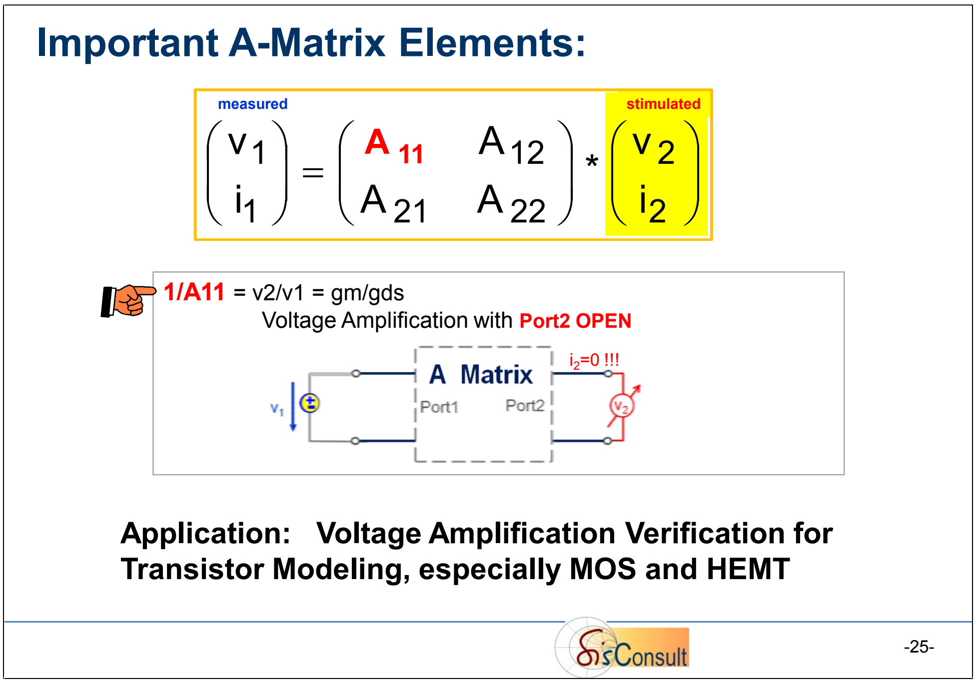
Notes /1/:

For reciprocal networks: A11*A22-A12*A21 = 1

For symmetrical networks: A11 = A22

For reciprocal and lossless networks, A11 and A22 are purely real, while A12 and A21 are purely imaginary.

/1/: Matthaei, Young, Jones, "Microwave Filters, Impedance-Matching Networks and Coupling Structures", McGraw-Hill, 1964



Notice:

1/A11 means v2/v1 with stimulus v2, but i2=0 !!!

To perform such a measurement, and to satisfy this condition with i2=0, Port1 has to be stimulated with v1, and v2 is measured at the open Port2.

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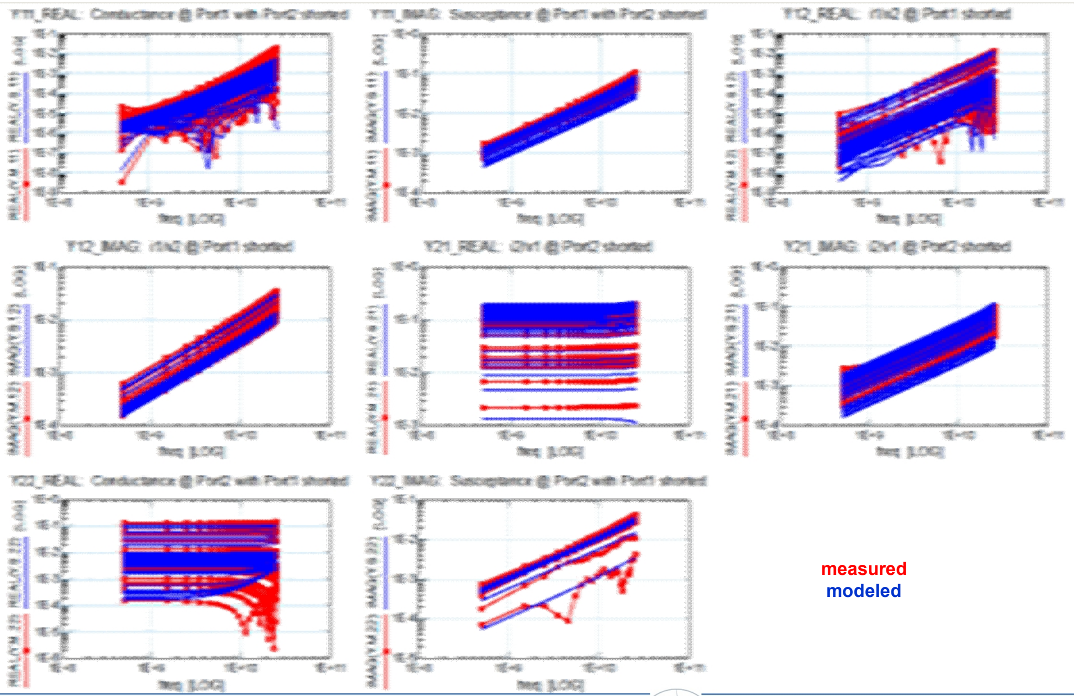
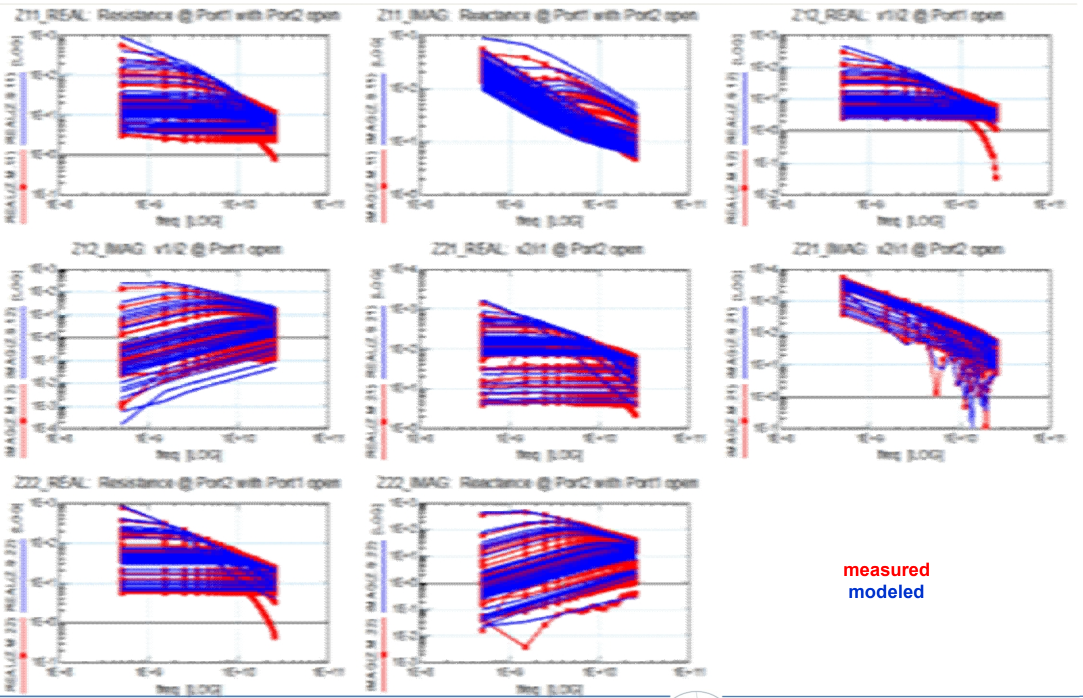
Examples of Applying the TwoPort Matrices for Verification of Spice Modeling Fit

Example: Angelov HEMT Modeling: Y-Parameter Fit

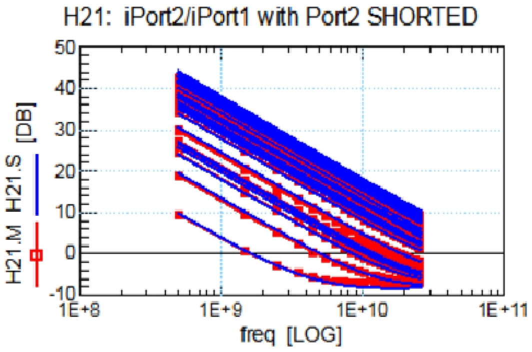


Example: Angelov HEMT Modeling: Z-Parameter Fit

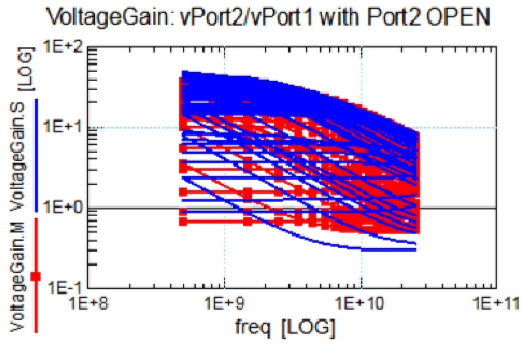


Example: Angelov HEMT Modeling: H21, 1/A11

H_{21}



$G_{21} = 1/A_{11} = gm/gds$



measured
modeled

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Wrap-Up

In a Nut Shell:
Circuit Characteristics Directly from Z, Y, H and A

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} * \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} \qquad \begin{pmatrix} v_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} * \begin{pmatrix} i_1 \\ v_2 \end{pmatrix}$$

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} * \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \qquad \begin{pmatrix} v_1 \\ i_1 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} * \begin{pmatrix} v_2 \\ i_2 \end{pmatrix}$$

Characteristics	Condition	Z	Y	H	A
Input Impedance	Port2 OPEN	Z11			
	Port2 SHORTED		1/Y11		
Output Impedance	Port1 OPEN	Z22			
	Port1 SHORTED		1/Y22		
Trans-Conductance (gm=i2/v1)	Port2 SHORTED		Y21		
Current Amplification i2/i1	Port2 SHORTED			H21	
Voltage Amplification v2/v1 = gm/gds	Port2 OPEN				1/A11

Thank You !

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